Chapter 15: Derivatives
What you need to KNOW

Big Idea	What you use	An example
Know the limit definition of a derivative	$\lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$	Find the formula for the slope of a secant line from time x to time x+Δx for the function $f(x) = 2x - 3$. Show work. $\lim\limits_{\Delta x \to 0} \frac{2(x+\Delta x)-3-(2x-3)}{\Delta x}$ $\lim\limits_{\Delta x \to 0} \frac{2x+2\Delta x-3-2x+3}{\Delta x}$ $\lim\limits_{\Delta x \to 0} \frac{2\Delta x}{\Delta x} = \boxed{2}$
Know some basic physics	If you see "average velocity from time a to time b", then that is the slope, the average rate of change between given points. Position $f(x)$ Instantaneous Velocity $f'(x)$ $\uparrow$ # Instantaneous Acceleration $f''(x)$ $\uparrow$ #	The height of an object at t seconds with initial velocity of 50 ft/sec is given by $h(t) = 50t - 16t^2$. a. find the average rate of change from time 2 seconds to 4 seconds $h(2) = 36$ $h(4) = -56$ $m = \frac{\Delta y}{\Delta x} = \frac{-56-36}{4-2} = \frac{-92}{2} = \boxed{-46 \text{ ft/sec}}$ b. What is the formula for instantaneous velocity of the object? Use the formula to find the velocity of the ball at 3 seconds. $h'(t) = -32t + 50$ $h'(3) = \boxed{-46 \text{ ft/sec}}$ c. What is the formula for the instantaneous acceleration of the object? Use the formula to find the acceleration at 3 seconds. $h''(t) = -32$ $h''(3) = \boxed{-32 \text{ ft./sec}^2}$ d. At what time does the object hit the ground? $h(t) = 0$ $50t - 16t^2 = 0$ $2t(25 - 8t) = 0$ $2t = 0$ $t = 0$ $25 - 8t = 0$ $\boxed{t = 3.125 \text{ sec.}}$ e. At what time does the object reach its maximum height? When deriv. = 0 $h'(t) = 0$ $-32t + 50 = 0$ $t = 1.5625$ $h(1.5625) = 39.0625$ max at 1.5625 sec. is 39.0625 ft.

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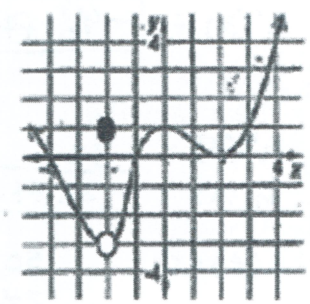
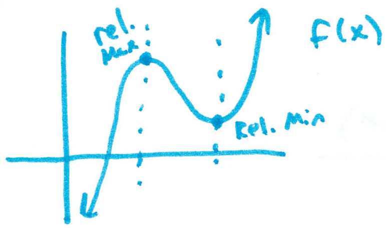
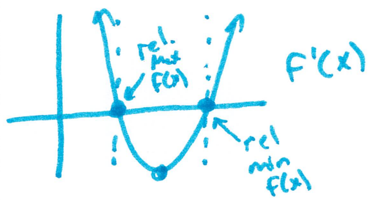
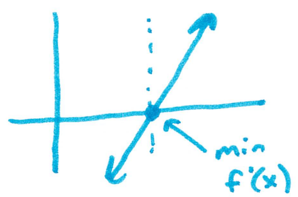
Write an equation for a tangent line for a function at a particular point	$f(x)$ ↑ replace $x = \#$ to get y $f'(x)$ ↑ replace x to get m $y - y_1 = m(x - x_1)$	Write the equation of the tangent line for the function $f(x) = 2x^3 - 3x^2 - 10x \quad at\ x = 3$. Show all work in determining this equation. $f(3) = 2(3)^3 - 3(3)^2 - 10(3)$ $= -3$ Point: $(3, -3)$ $m = 26$ $f'(x) = 6x^2 - 6x - 10$ $f'(3) = 6(3)^2 - 6(3) - 10$ $= 26$ $\boxed{y + 3 = 26(x - 3)}$
Read a graph to answer some basic questions about limits and rate of change		Given the function graphed below, find $f(-2)$ and $\lim_{x \to -2} f(x)$. Is $f(x)$ continuous at $x = -2$? ↓ $f(-2) = 1$ $\lim_{x \to -2} f(x) = -3$ Not continuous  Find the average rate of change from x = -3 to x = 3. $(-3, -2)$ $(3, 1)$ $\frac{\Delta y}{\Delta x} = \frac{3}{6} = \boxed{\frac{1}{2}}$

<table>
<tr><th>Big Idea</th><th>What you use</th><th>An example</th></tr>
<tr><td>Derivative basics</td><td>






</td><td>Using your knowledge of derivatives, answer the following questions:
a. What is a tangent line and what does it tell you?
Instantaneous velocity @ a given time.
b. What is a secant line?
Average velocity between 2 times (2 points)
c. What is instantaneous velocity and how do you find it?
Slope of tangent line (find $f'(x)$ & plug in x)
d. What is instantaneous acceleration and how do you find it?
2nd derivative
e. The derivative function is really the <u>slope</u> function of the original function.
f. The <u>zeros</u> of the derivative are the <u>rel. max/rel. min</u> points of the original function.

Given the function $f(x) = 6x^7 - 9x^4 + 3x^2 + 2$, find $f'(x)$ and $f''(x)$.
$f'(x) = 42x^6 - 36x^3 + 6x$
$f''(x) = 252x^5 - 108x^2 + 6$</td></tr>
<tr><td>Product Rule</td><td>1d2 + 2d1</td><td>$f(x) = (2x-4)\sin x$
$f'(x) = (2x-4)(\cos x) + (\sin x)(2)$
$= 2x\cos x - 4\cos x + 2\sin x$</td></tr>
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Big Idea	What you use	An example
Quotient Rule	$\dfrac{\text{low(dhigh)} - \text{high(dlow)}}{\text{denom}^2}$	$f(x) = \dfrac{2x-7}{e^x}$ $f'(x) = \dfrac{(e^x)(2) - (2x-7)(e^x)}{(e^x)^2}$ $= \dfrac{2e^x - 2xe^x + 7e^x}{(e^x)^2}$ $= \boxed{\dfrac{-2x+9}{e^x}}$
~~Product and Quotient Rule~~	**Skip this row**	$\cancel{y = \dfrac{xe^x}{x^2+2}}$

Big Idea	What you use	An example
Chain Rule	$u' \cdot f'(u)$	$f(x) = \cos(x^2 - 4)$ $u = x^2 - 4$ $\quad f(u) = \cos u$ $u' = 2x$ $\quad f'(u) = -\sin u$ $f'(x) = (2x)(-\sin(x^2-4))$ $= \boxed{-2x\sin(x^2-4)}$ $f(x) = \ln(5x^3 - 2x + 8)$ $u = 5x^3 - 2x + 8$ $\quad f(u) = \ln u$ $u' = 15x^2 - 2$ $\quad f'(u) = \frac{1}{u}$ $f'(x) = (15x^2-2)\left(\frac{1}{5x^3-2x+8}\right)$ $= \boxed{\frac{15x^2-2}{5x^3-2x+8}}$