Section 5.1: Probability and Odds

Objectives:
- Find the probability of an event
- Find the odds for the success and failure of an event.

Definition: Success – desired outcome

Definition: Failure - an outcome other than the desired outcome

Definition: If an event can succeed in *s* ways and fail in *f* ways, then the probability of success *P*(*s*) and the probability of failure *P*(*f*) are as follows:

$$p(s) = \frac{s}{s+f} \qquad p(f) = \frac{f}{s+f} = 1 - p(s)$$

Example 1:
To determine tv ratings, Nielsen Media Research estimates how many people are watching any given television program. This is done by selecting a sample audience, having them record their viewing habits in a journal, and then counting the number of viewers for each program. There are about 100 million households in the US and only 5000 are selected for the sample group. What is the probability of any one household being chosen to participate for the Nielson Media Research Group?

$$s = 5000 \qquad s+f = 100{,}000{,}000$$

$$p(s) = \frac{5000}{100{,}000{,}000} = \frac{1}{20{,}000} = .005\%$$

Example 2:
A bag contains 5 yellow, 6 blue, and 4 white marbles.

a. What is the probability that a marble selected at random will be yellow?

$$p(\text{yellow}) = \frac{\text{yellow}}{\text{yell} + \sim\text{yell}} = \frac{5}{5+10} = \frac{1}{3}$$

b. What is the probability that a marble selected at random will *not* be white?

$$p(\sim\text{white}) = \frac{\sim\text{white}}{\sim\text{white} + \text{white}} = \frac{11}{11+4} = \frac{11}{15}$$

Example 3:
A circuit board with 20 computer chips contains 4 chips that are defective. If 3 chips are selected at random, what is the probability that all 3 are defective?

$$\frac{{}_4C_3}{{}_{20}C_3} \begin{array}{l} \leftarrow \text{ways to select 3 defective chips} \\ \leftarrow \text{ways to select 3 chips} \end{array}$$

$$= \frac{1}{285}$$

Definition: Complements – $1 - P(s) = \text{complement}$

$p(f)$ and $p(s)$ are complements!

Example 4:
The CyberToy Company has determined that out of a production run of 50 toys, 17 are defective. If 5 toys are chosen at random, what is the probability that at least 1 is defective?

$P(\text{at least 1 defective}) = 1 - P(\text{no defective})$

$$1 - \frac{{}_{33}C_5}{{}_{50}C_5} \begin{array}{l} \leftarrow \text{ways to choose 5 non-defective toys} \\ \leftarrow \text{ways to choose 5 toys} \end{array}$$

$= .8879835375$

Definition: Odds –

Ratio of $p(s)$ to $p(f)$

$$\text{Odds} = \frac{P(s)}{P(f)}$$

Example 5:

Katrina must select at random a chip from a box to determine which question she will receive in a math contest. There are 6 blue and 4 red chips in the box. If she selects a blue chip, she will have to solve a trig problem. If the chip is red, she will have to write a geometry proof.

a. What is the probability that Katrina will draw a red chip?

$$\frac{4}{10} = \frac{2}{5}$$

b. What are the odds that Katrina will have to write a geometry proof?

$$P(s) = \frac{2}{5}$$

$$P(f) = \frac{3}{5}$$

$$\text{Odds} = \frac{2/5}{3/5} = 2/3$$

Example 6:

Twelve male and 16 female students have been selected as equal qualifiers for 6 college scholarships. If the awarded recipients are to be chosen at random, what are the odds that 3 will be male and 3 will be female?

${}_{12}C_3 \leftarrow$ # of groups of 3 males

${}_{16}C_3 \leftarrow$ # of groups of 3 females

${}_{12}C_3 \cdot {}_{16}C_3 = 123{,}200$ possible groups

Total Qualified candidates: ${}_{28}C_6 = 376{,}740$

# of groups without 3 boys/3 girls: $376{,}740 - 123{,}200 = 253{,}540$

$$P(s) = \frac{123{,}200}{376{,}740} \qquad P(f) = \frac{253{,}540}{376{,}740}$$

$$\text{Odds: } \frac{123{,}200}{253{,}540} = \frac{880}{1811} = .4859$$

Section 5.2: Probabilities of Compound Events

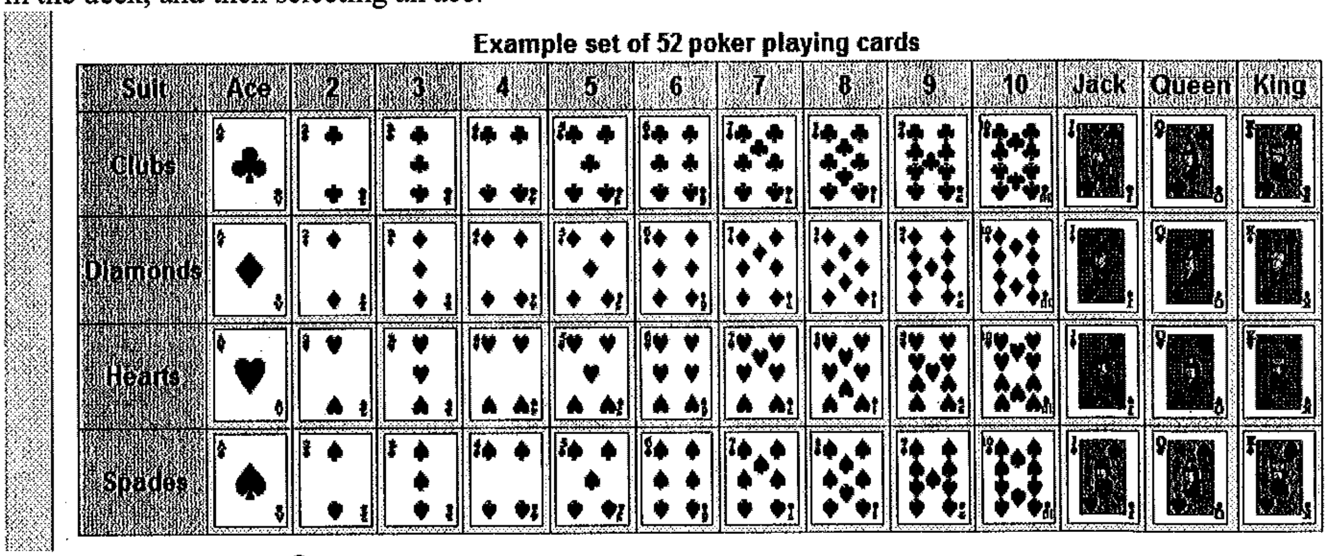
Objectives:

- Find the probability of independent and depended events.
- Identify mutually exclusive events.
- Find the probability of mutually exclusive and inclusive events.

Definition: If two events, A and B, are independent, then the probability of both events occurring is the product of each individual probability:

P(A and B) = P(A) * P(B)

Example 1:
Using a standard deck of playing cards, find the probability of selecting a face card, replacing it in the deck, and then selecting an ace.



$P(A) = \frac{12}{52}$

$P(B) = \frac{4}{52}$

$\frac{12}{52} \cdot \frac{4}{52} = \frac{3}{169}$

Example 2:
Statistics collected in a particular coal-mining region show that the probability that a miner will develop black lung disease is 5/11. Also, the probability that a miner will develop arthritis is 1/5. If one health problem does not affect the other, what is the probability that a randomly-selected miner will not develop black lung disease but will develop arthritis?

P(~black lung and arthritis) = [1 − P(black lung)] · P(arthritis)

$(1 - 5/11) \cdot \frac{1}{5} = \frac{6}{55}$

Definition: If two events, A and B, are dependent, then the probability of both events occurring is the product of each individual probability:

P(A and B) = P(A) * P(B following A)

Example 3:

Tasha has 3 rock, 4 country, and 2 jazz CD's in her car. One day, before she starts driving, she pulls 2 CD's from her CD carrier without looking.

a. Determine if the events are independent or dependent.

dependent

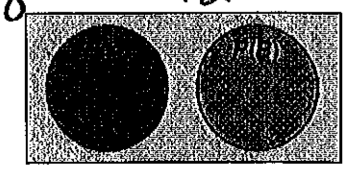
b. What is the probability that both CD's are rock?

P(rock | rock) = P(rock) · P(rock following rock)

$\frac{3}{9} \cdot \frac{2}{8} = \frac{1}{12}$

Definition: Mutually Exclusive –

when 2 events cannot happen at the same time.



Events A and B are mutually exclusive.

Definition: If two events, A and B, are mutually exclusive, then the probability that either A or B occurs is the sum of their probabilities:

P (A or B) = P(A) + P(B)

Example 4:

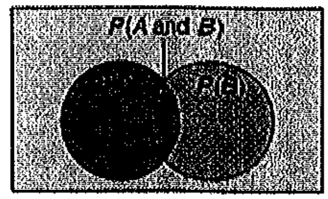
Leonard is a contestant in a game where if he selects a blue ball or a red ball he gets an all-expenses paid Caribbean cruise. Leonard must select the ball at random from a box containing 2 blue, 3 red, 9 yellow, and 10 green balls. What is the probability that he will win the cruise?

P(blue or red) = P(blue) + P(red)

$\frac{2}{24} + \frac{3}{24} = \frac{5}{24}$

Definition: Inclusive –

events that are not mutually exclusive



Events A and B are inclusive events.

Definition: If two events, A and B, are inclusive, then the probability that either A or B occurs is the sum of their probabilities decreased by the probability of both occurring.

P (A or B) = P(A) + P(B) – P(A and B)

Example 5:

Kerry has read that the probability for a driver's license applicant to pass the road test the first time is 5/6. He has also read that the probability of passing the written examination on the first attempt is 9/10. The probability of passing both the road and written examinations on the first attempt is 4/5.

a. Determine if the events are mutually exclusive or inclusive.

Because you can pass both exams, inclusive.

b. What is the probability that Kerry can pass either examination on his first attempt?

P(road) + P(written) – P(both)

$$\frac{5}{6} + \frac{9}{10} - \frac{4}{5} = \frac{56}{60} = \frac{14}{15}$$

Example 6:

There are 5 students and 4 teachers on the school publications committee. A group of 5 members is being selected at random to attend a workshop on school newspapers. What is the probability that the group attending the workshop will have at least 3 students?

At least 3 = 3, 4, or 5 (mutually exclusive)

P(3) + P(4) + P(5)

$$\frac{{}_5C_3 \cdot {}_4C_2}{{}_9C_5} + \frac{{}_5C_4 \cdot {}_4C_1}{{}_9C_5} + \frac{{}_5C_5 \cdot {}_4C_0}{{}_9C_5}$$

$$= \frac{9}{14}$$

Section 5.3: Conditional Probability

Objectives:

- Find the probability of an event given the occurrence of another event.

Definition: The conditional probability of event A, given event B, is defined as

$$P(A \mid B) = \frac{P(AandB)}{P(B)} \text{ where } P(B) \neq 0.$$

Example 1:

Danielle Jones works in a medical research laboratory where a drug that promotes hair growth in balding men is being tested. The results of the preliminary tests are shown in the table below. What is the probability that a test subject's hair grew, given that he used the experimental drug?

$$P(H \mid D) = \frac{P(\text{grew hair \& used drug})}{P(\text{used drug})}$$

	Number of Subjects		
	Using Drug	Using Placebo	
Hair growth	1600	1200	2800
No hair growth	800	400	1200
	2400	1600	4000

$$\frac{\frac{1600}{4000}}{\frac{2400}{4000}} = \frac{1600}{2400} = \frac{2}{3}$$

Example 2:

Denette tosses two coins. What is the probability that she has tossed 2 heads, given that she has tossed at least 1 head?

Sample Space: {HH, HT, TH, TT}

A= 2 heads

B= at least one head

$P(A \cap B) = \frac{1}{4}$

$P(B) = \frac{3}{4}$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{3/4} = 1/3$$

Example 3:
Alfonso is conducting a survey of families with 3 children. If a family is selected at random, what is the probability that the family will have exactly 2 boys if the second child is a boy? Assume that the probability of giving birth to a boy is equal to the probability of giving birth to a girl.

Sample Space: {BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG}

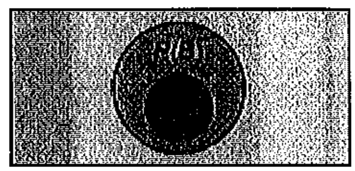
X = 2 boys $P(X) = 3/8$

y = 2nd child is a boy $P(y) = 4/8$

$P(X \cap Y) = 2/8$ {BBG, GBB}

$$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{2/8}{4/8} = \boxed{\frac{1}{2}}$$

Definition: If A is a subset of B, then $P(A \mid B) = \dfrac{P(A)}{P(B)}$.

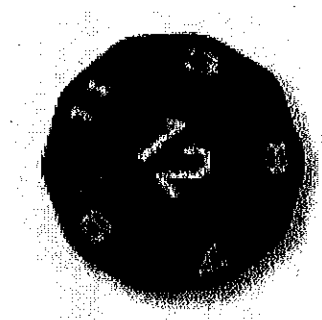


Event A is a subset of event B.

Example 4:
A 12-sided dodecahedron has the numerals 1 through 12 on its faces. The die is rolled once, and the number on the top face is recorded. What is the probability that the number is a multiple of 4 if it is known that it is even?

A = {4, 8, 12} B = {2, 4, 6, 8, 10, 12}

$P(A) = 3/12$ $P(B) = 6/12$



$P(A \cap B) = P(A)$ b/c A is a subset of B.

$$P(A|B) = \frac{P(A)}{P(B)} = \frac{1/4}{1/2} = \frac{1}{2}$$

Section 5.4: The Binomial Theorem and Probability

Objectives:

- Find the probability of an event by using the Binomial Theorem.

Example 1:

Managers at the Eco-Landscaping Company know that a mahogany tree they plant has a survival rate of about 90% if cared for properly. Let S = probability of a tree surviving and D = probability of a tree dying.

a. If 10 trees are planted in that last phase of a landscaping project, what is the probability that 7 of the trees will survive?

Binomial term with s^7d^3

$p(S) = .90$

$p(d) = .10$

${}_{10}C_3 s^7d^3 = 120 s^7d^3$

$120(.9)^7(.1)^3 = .057395628$

$\approx 5.7\%$ chance

b. If 5 trees are planted, what is the probability that at least 2 trees die?

$1 - P(0 \text{ or } 1)$

$1 - \left({}_5C_0(.9)^5(.1)^0 + {}_5C_1(.9)^4(.1)^1\right) = .0815 \approx 8.15\%$

Definition: Conditions of a Binomial Experiment –

A binomial experiment exists if and only if these conditions occur.

- Each trial has exactly two outcomes, or outcomes that can be reduced to two outcomes.
- There must be a fixed number of trials.
- The outcomes of each trial must be independent.
- The probabilities in each trial are the same.

Example 2:

Eight out of every ten persons who contract a certain viral infection can recover. If a group of 7 people become infected, what is the probability that exactly 3 people will recover from the infection?

R^3N^4

$P(R) = .80$

$P(N) = .20$

${}_7C_4 R^3N^4$

$35(.8)^3(.2)^4$

$= .028672 \approx 2.87\%$

Example 3:

Refer back to example 1. What is the probability that at least 7 of the 10 trees planted will survive?

$${}_{10}C_3\, s^7 d^3 + {}_{10}C_2\, s^8 d^2 + {}_{10}C_1\, s^9 d^1 + {}_{10}C_0\, s^{10} d^0$$

$$120(.9)^7(.1)^3 + 45(.9)^8(.1)^2 + 10(.9)^9(.1)^1 + 1(.9)^{10}$$

$$= 0.987204$$

$$\approx 98.7\%$$

Example 4:

Of the computer operators who work for a large temporary employment agency, 60% have a car. If 5 of the computer operators are randomly chosen to work on a job that requires car transportation, what is the probability that at least 4 have cars?

$${}_5C_1\, c^4 n^1 + {}_5C_0\, c^5 n^0$$

$$5(.6)^4(.4)^1 + 1\cdot(.6)^5\cdot 1$$

$$= .33696 \approx 33.7\%$$

Section 5.5: Expected Value

Objectives:

- Find expected value in situations involving gains and losses.
- Determine whether a game is fair.

In this section, we will explore probabilistic situations involving gains or losses.
For example:

1. What can you expect to win or lose in various games of chance?
2. What is the value of a $1 ticket in a $1 million lottery?
3. Should someone who is 18 years old pay $160 for collision damage insurance on his/her $1500 car?

Let's explore the first situation:
Consider a simple game in which a die is rolled and you win points from, or lose points to, another player as follows.

Event	**Die shows 1, 2, or 3**	**Die shows 4 or 5**	**Die shows 6**
Gain or Loss	+ 10 points	-13 points	-1 point
Probability	½	1/3	1/6

To calculate the **expected value** of this game, you must multiply each gain or loss by its probability and then add the products together. What is the expected value of this game?

$10(1/2) + (-13)(1/3) + (-1)(1/6) = 0.5$

Meaning of this expected value? the average gain per game is 0.5.

If the **expected value** of a game is 0, then the game is called a fair game.

Example 1:
If the sum of two rolled dice is 8 or more, you win $2; if not, you lose $1.

a. Show that this is not a fair game.
b. To have a fair game, the $2 winnings should instead be what amount?

$\frac{8}{5}$ $\frac{9}{4}$ $\frac{10}{3}$ $\frac{11}{2}$ $\frac{12}{1}$

Event	Sum ≥ 8	sum < 8
Prob.	15/36	21/36
Payoff	$2	$1

a) $\frac{15}{36}(2) + \frac{21}{36}(-1) = \frac{1}{4}$

b) $\frac{15}{36}x - \frac{21}{36} = 0$

$\frac{15}{36}x = \frac{21}{36}$

$x = \$1.40$

Example 2:

In a certain state's lottery, six numbers are randomly chosen without repetition from the numbers 1 to 40. If you correctly pick all 6 numbers, only 5 of the 6, or only 4 of the 6, then you win $1 million, $1000, or $100 respectively. What is the value of a $1 lottery ticket?

Event	choose 6	choose 5	choose 4	choose 0,1,2,3
Prob.	$\frac{1}{3838380}$	$\frac{204}{3838380}$	$\frac{8415}{3838380}$	$\frac{3829760}{3838380}$
Payoff	$1,000,000	$1000	$100	$0

$P(\text{all 6 correct}) = \frac{{}_6C_6}{{}_{40}C_6} = \frac{1}{3838380}$

$P(\text{5 correct}) = \frac{{}_6C_5 \cdot {}_{34}C_1}{{}_{40}C_6} = \frac{204}{3838380}$

$P(\text{4 correct}) = \frac{{}_6C_4 \cdot {}_{34}C_2}{{}_{40}C_6} = \frac{8415}{3838380}$

Expected value:

$\frac{1000000}{3838380} + \frac{204000}{3838380} + \frac{841500}{3838380}$

$= \frac{2045500}{3838380} = .5329$

$\approx \$0.53$ for every $1 so loss (-$0.47)

Example 3:

An 18 year old student must decide whether to spend $160 for one year's car collision damage insurance. The insurance carries a $100 deductible, which means that when the student files a damage claim, the student must pay $100 of the damage amount, with the insurance company paying the rest (up to the value of the car). Because the car is only worth $1500, the student consults with an insurance agent who draws up a table of possible damage amounts and their probabilities based on the driving records for 18 year olds in the region.

Event	Accident costing $1500	Accident costing $1000	Accident costing $500	No accident
Payoff	$1400	$900	$400	$0
Probability	0.05	0.02	0.03	0.9

What is the expected value of this insurance?

Expected payoff $= 1400(0.05) + 900(0.02) + 400(0.03) + 0(.9)$

$= 70 + 18 + 12 + 0 = \$100$

Expected value $= 100 - 160 = -\$60$

loss of $60.00