Section 5.1: Probability and Odds **Name** Key

1. Using a standard deck of 52 cards, find each probability.
   a. P(face card)
   $\frac{4+4+4}{52} = \frac{3}{13}$
   b. P(a black, non-face card)
   $\frac{10+10}{52} = \frac{5}{13}$

2. One flower is randomly taken from a vase containing 5 red flowers, 2 white flowers, and 3 pink flowers. Find each probability.
   a. P(red)
   $\frac{5}{10} = \frac{1}{2}$
   b. P(not pink)
   $\frac{7}{10}$

3. Jacob has 10 rap, 18 rock, 8 country, and 4 pop MP3's in his music collection. Two are selected at random. Find each probability.
   a. P(2 pop)
   $\frac{{}_4C_2}{{}_{40}C_2} = \frac{1}{130}$
   b. P(1 rap and 1 rock)
   $\frac{{}_{10}C_1 \cdot {}_{18}C_1}{{}_{40}C_2} = \frac{3}{13}$

4. A number cube is thrown two times. What is the probability of rolling 2 fives?
   $\frac{1}{36}$

5. A box contains 1 green, 2 yellow, and 3 red marbles. Two marbles are drawn at random without replacement. What are the **odds** of each event occurring?
   a. Drawing 2 red marbles
   $P(S) = \frac{{}_3C_2}{{}_6C_2} = \frac{1}{5}$ $P(f) = \frac{4}{5}$ $\text{odds} = \frac{1/5}{4/5} = \frac{1}{4}$
   b. Not drawing yellow marbles.
   $P(S) = \frac{{}_4C_2}{{}_6C_2} = \frac{2}{5}$ $P(f) = 3/5$ $\text{odds} = \frac{2/5}{3/5} = 2/3$
   c. Drawing 1 green and 1 red.
   $P(S) = \frac{{}_1C_1 \cdot {}_3C_1}{{}_6C_2} = \frac{1}{5}$ $P(f) = 4/5$ $\text{odds} = \frac{1/5}{4/5} = 1/4$
   Skip! ~~d.~~ Drawing 2 different colors.
   $P(S) = \frac{{}_1C_1 \cdot {}_2C_1 + {}_1C_1 \cdot {}_3C_1 + {}_2C_1 \cdot {}_3C_1}{{}_6C_2} = \frac{11}{15}$ $\text{odds} = \frac{11/15}{4/15} = 11/4$

6. The odds of winning a prize in a raffle with one raffle ticket are $\frac{1}{249}$. What is the probability of winning with one ticket?
   $\frac{1/x}{249/x}$ $x = 250$ $P(S) = 1/250$

7. During a particular hurricane, hurricane trackers determine that the odds of it hitting the South Carolina coast are 1 to 4. What is the probability of this event happening?

$$\frac{1/x}{4/x} \quad x=5 \qquad P(S)=1/5$$

8. Kim uses a combination lock on her locker that has 3 wheels, each labeled with 10 digits from 0 to 9. The combination is a particular sequence with no digits repeating.
   a. What is the probability of someone guessing the correct combination?

$$P(S)=\frac{1}{10}\cdot\frac{1}{9}\cdot\frac{1}{8}=\frac{1}{720}$$

   b. If the digits can be repeated, what are the odds against someone guessing the combination?

$$P(f)=\frac{1}{10^3}=\frac{1}{1000}$$

$$P(S)=\frac{999}{1000} \quad \text{odds } \frac{999/1000}{1/1000}=999/1$$

9. Mrs. Meyer gives her Discrete math class 20 review problems. She will select 10 of them to answer on an upcoming test. Hannah can correctly solve 15 of the problems.
   a. Find the probability that Hannah can correctly solve all 10 problems on the test.

$$\frac{{}_{15}C_{10}}{{}_{20}C_{10}}=\frac{3003}{184756}=21/1292$$

   b. Find the odds that Hannah will know how to solve 8 of the 10 problems. correctly.

$$P(S)=\frac{{}_{15}C_8\cdot{}_5C_2}{{}_{20}C_{10}}=\frac{225}{646} \qquad P(f)=\frac{421}{646}$$

$$\text{Odds}: \frac{225}{421}$$

Section 5.2: Probabilities of Compound Events

1. Determine if each event is *independent* or *dependent*. Then determine the probability.
   a. The probability of selecting a blue marble, not replacing it, then a yellow marble from a box of 5 blue marbles and 4 yellow marbles.

   dependent ;

   $$\frac{5}{9} \cdot \frac{4}{8} = \frac{20}{72} = \frac{5}{18}$$

   b. A green number cube and a red number cube are tossed. What is the probability that a 4 is shown on the green number cube and a 5 is shown on the red number cube?

   independent ;

   $$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

   c. A bank contains 4 nickels, 4 dimes, and 7 quarters. Three coins are removed in sequence, without replacement. What is the probability of selecting a nickel, a dime, and a quarter in that order?

   dependent ;

   $$\frac{4}{15} \cdot \frac{4}{14} \cdot \frac{7}{13} = \frac{8}{195}$$

   d. The probability of randomly selecting a knife, a fork, and a spoon in that order from a kitchen drawer containing 8 spoons, 8 forks, and 12 knives.

   dependent ;

   $$\frac{12}{28} \cdot \frac{8}{27} \cdot \frac{8}{26} = \frac{32}{819}$$

   e. The probability that a football team will win its next four games if the odds of winning each game are 4:3.

   $$\text{Odds} = \frac{4/7}{3/7} \qquad P(S) = \frac{4}{7}$$

   $$P(\text{winning } 4) = \left(\frac{4}{7}\right)^4 = \frac{256}{2401}$$

2. Determine if each event is *mutually exclusive* or *mutually inclusive*. Then determine each probability.

a. The probability of selecting an ace or a red card from a standard deck of cards.

inclusive;

$$\frac{4}{52}+\frac{26}{52}-\frac{2}{52}=\frac{7}{13}$$

b. The probability of randomly picking 5 puppies of which at least 3 are male puppies, from a group of 5 male puppies and 4 female puppies.

exclusive; P(at least 3m) = P(3m/2f) + P(4m/1f) + P(5m)

$$\frac{{}_5C_3 \cdot {}_4C_2}{{}_9C_5}+\frac{{}_5C_4 \cdot {}_4C_1}{{}_9C_5}+\frac{{}_5C_5 \cdot {}_4C_0}{{}_9C_5}=\frac{9}{14}$$

c. The probability that a group of 6 people selected at random from 7 men and 7 women will have at least 3 women. exclusive

$1-(P(0)+P(1)+P(2))$

$$1-\left(\frac{{}_7C_0 \cdot {}_7C_6}{{}_{14}C_6}+\frac{{}_7C_1 \cdot {}_7C_5}{{}_{14}C_6}+\frac{{}_7C_2 \cdot {}_7C_4}{{}_{14}C_6}\right)=\frac{302}{429}$$

d. The probability that two cards drawn from a standard deck will both be aces or both will be black.

inclusive

$$\left(\frac{4}{52}\cdot\frac{3}{51}\right)+\left(\frac{26}{52}\cdot\frac{25}{51}\right)-\left(\frac{2}{52}\cdot\frac{1}{51}\right)$$

$$=\frac{55}{221}$$

3. There are six women and seven men on a committee for city services improvement. A subcommittee of five members is being selected at random to study the feasibility of modernizing the water treatment facility. What is the probability that the committee will have at least three women?

P(3W) + P(4W) + P(5W)

$$\frac{{}_6C_3 \cdot {}_7C_2}{{}_{13}C_5}+\frac{{}_6C_4 \cdot {}_7C_1}{{}_{13}C_5}+\frac{{}_6C_5 \cdot {}_7C_0}{{}_{13}C_5}$$

$$=\frac{59}{143}$$

Section 5.3: Conditional Probability

Find each probability.

1. Two coins are tossed. What is the probability that one coin shows heads if it is known that at least one coin is tails?

$$P(\text{head} \mid \overset{\text{at least}}{\text{1 tail}}) = \frac{2/4}{3/4} = \frac{2}{3}$$

2. A city council consists of 6 Democrats, two of whom are women, and six are Republicans, four of whom are men. A member is chosen at random. If the member chosen is a man, what is the probability that he is a Democrat?

$$P(\text{dem} \mid \text{man}) = \frac{4/12}{8/12} = 1/2$$

3. A bag contains 4 red chips and 4 blue chips. Another bag contains 2 red chips and 6 blue chips. A chip is randomly selected from one of the bags, and found to be blue. What is the probability that the chip is from the first bag?

$$P(\text{1st bag} \mid \text{1st chip blue}) = \frac{4/16}{10/16} = 2/5$$

4. Two boys and two girls are lined up at random. What is the probability that the girls are separated if a girl is at an end?

$$P(\text{girls sep} \mid \text{girl @ end}) = \frac{3/6}{5/6} = 3/5$$

Skip → 5. A five-digit number is formed from the digits 1, 2, 3, 4, and 5. What is the probability that the number ends in the digits 52, given that it is even?

$$P(\text{end in 52} \mid \text{even}) = \frac{\frac{3\cdot2\cdot1}{5!}}{\frac{4!}{5!}+\frac{4!}{5!}} = 1/8$$

6. Two game tiles, numbered 1 through 9, are selected at random from a box without replacement. If their sum is even, what is the probability that both numbers are odd?

$$P(\text{2 odd} \mid \text{sum even}) = \frac{20/72}{32/72} = 5/8$$

7. A container holds 3 green marbles and 5 yellow marbles. One marble is randomly drawn and discarded. Then a second marble is drawn. Find each probability.

   a. The second marble is green, given that the first marble was green.

   $$P(\text{2nd green} \mid \text{1st green}) = \frac{\cancel{\frac{3}{8}} \cdot \frac{2}{7}}{\cancel{\frac{3}{8}}} = \frac{2}{7}$$

   b. The second marble is yellow, given that the first marble was green.

   $$P(\text{2nd yellow} \mid \text{1st green}) = \frac{\cancel{\frac{3}{8}} \cdot \frac{5}{7}}{\cancel{\frac{3}{8}}} = \frac{5}{7}$$

   c. The second marble is yellow, given that the first marble was yellow.

   $$P(\text{2nd yellow} \mid \text{1st yellow}) = \frac{\cancel{\frac{5}{8}} \cdot \frac{4}{7}}{\cancel{\frac{5}{8}}} = \frac{4}{7}$$

8. In a game played with a standard deck of cards, each face card has a value of 10 points, each ace has a value of 1 point, and each number card has a value equal to its number. Two cards are drawn at random.

   a. At least one card is an ace. What is the probability that the sum of the cards is 7 or less?

   $A = \text{sum} \leq 7$
   $B = \text{at least one ace}$
   $B' = \text{no aces}$

   $$P(B') = \frac{{}_{48}C_2}{{}_{52}C_2} = \frac{188}{221}$$

   $$P(B) = 1 - \frac{188}{221} = \frac{33}{221}$$

   $$P(A \cap B) = \frac{{}_4C_2 + {}_4C_1 + {}_{20}C_1}{{}_{52}C_2} = \frac{43}{663}$$

   $$P(A \mid B) = \frac{43/663}{33/221} = \frac{43}{99}$$

   b. One card is the queen of diamonds. What is the probability that the sum of the cards is greater than 18?

   $$P(\text{sum} > 18 \mid \text{queen } \Diamond\text{'s}) = \frac{\cancel{\frac{1}{52}} \cdot \frac{19}{51}}{\cancel{\frac{1}{52}}} = \frac{19}{51}$$

9. In a game using two number cubes, a sum of 10 has not turned up in the past few rolls. A player believes that a roll of 10 is "due" to come up. Analyze the player's thinking.

   The probability of rolling a sum of 10 remains the same (independent of all rolls before it).

Section 5.4: The Binomial Theorem and Probability

1. Midge carries lipstick tubes in a bag in her purse. The probability of pulling out the color she wants is 2/3. Suppose she uses her lipstick 4 times in a day. Find each probability.
   a. P(never the correct color)

   $4C_4 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^4 = \frac{1}{81}$

   b. P(no more than 3 times correct)

   $1 - P(4 \text{ correct})$

   $1 - \left(4C_0 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^0\right) = \frac{65}{81}$

2. Amber guesses at all 10 questions on a true/false test. Find each probability.
   a. P(7 correct)

   $10C_3 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3 = \frac{15}{128}$

   b. P(all correct)

   $10C_0 \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0 = \frac{1}{1024}$

3. The probability of tossing a head on a bent coin is 1/3. Find each probability if the coin is tossed 4 times.
   a. P(4 heads)

   $4C_0 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^0 = \frac{1}{81}$

   b. P(at least 2 heads)

   $4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2 + 4C_1 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^1 + 4C_0 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right.$

   $= 11/27$

4. If a thumbtack is dropped, the probability of its landing point up is 2/5. Miss Kemperman drops 10 tacks while putting up the weekly assignment sheet on the bulletin board. Find each probability.
   a. P(all point up)

   $10C_0 \left(\frac{2}{5}\right)^{10} \left(\frac{3}{5}\right)^0$

   $= 1.049 \times 10^{-4}$

   b. P(exactly 5 point up)

   $10C_5 \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^5 = .201$

5. Find each probability if 3 coins are tossed.
   a. P(3 heads or 3 tails)

   $3C_0 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0 + 3C_3 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^3$

   $= \frac{1}{4}$

   b. P(exactly 2 tails)

   $3C_2 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2 = 3/8$

6. During the Gulf War in 1990 – 1991, SCUD missiles hit 20% of their targets. In one incident, six missiles were fired at a fuel storage installation.
   a. Describe what success means in this case, and state the number of trials and the probability of success on each trial.

      success = missile hits target ; 6 trials

      $P(S) = 20\%$ or $1/5$

   b. Find the probability that between 2 and 6 missiles hit the target, inclusive.

      $1 - (P(0) + P(1))$

      $1 - \left({}_6C_6\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^6 + {}_6C_5\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^5\right) = \frac{1077}{3125}$

7. Door prizes are given at a party through a drawing. Four out of 10 tickets are given to men who will attend, and 6 out of 10 tickets are distributed to women. Each person will receive only one ticket. Ten tickets will be drawn at random with replacement. What is the probability that all winners will be the same sex?

   P(all men or all women)

   $${}_{10}C_0\left(\frac{4}{10}\right)^{10}\left(\frac{6}{10}\right)^0 + {}_{10}C_{10}\left(\frac{4}{10}\right)^0\left(\frac{6}{10}\right)^{10}$$

   $= .00615$

Section 5.5: Expected Value

1. Find the expected payoff using the table below:

Payoff	9	7	-5
Probability	0.1	0.3	0.6

$9(0.1) + 7(0.3) + -5(0.6) = 0$

$.9 + 2.1 - 3.0$

2. Find the expected payoff using the table below:

Payoff	6	3	-5
Probability	0.2	0.1	0.7

$6(0.2) + 3(0.1) - 5(0.7)$

$1.2 + .3 - 3.5 = -2$

3. For each scenario below, decide if the game is a fair game. If not, state which player has the advantage.

   a. A die is rolled. If the number that shows is odd, player A wins \$1 from player B. If it is a 6, A wins \$2 from player B. Otherwise B wins \$3 from A.

Odd	6	2, 4
3/6	1/6	2/6
\$1	\$2	-\$3

$(1)3/6 + 1/6(2) - 3(2/6) = -1/6$

Player B has the advantage.

   b. Two dice are rolled. If the sum of the numbers showing on the dice is odd, player A wins \$1 from player B. If both dice show the same number, A wins \$3 from B. Otherwise B wins \$3 from A.

Odd	Doubles	Other
1/2	6/36	12/36
\$1	\$3	-\$3

$\frac{1}{2} + (3)6/36 - (3)12/36$

$\frac{1}{2} + \frac{1}{2} - 1 = 0$ fair game

4. A box contains 3 red marbles and 2 ~~red~~ green marbles. ~~Tow~~ Two marbles are randomly chosen without replacement. If both are green, you win \$2. If just one is green, you win \$1. Otherwise, you lose \$1. What is your expected gain or loss?

2 green	1 green	No green
1/10	6/10	3/10
\$2	\$1	-\$1

$P(2\text{ green}) = \frac{{}_2C_2}{{}_5C_2} = \frac{1}{10}$

$P(1\text{ green } \& \ 1\text{ red}) = \frac{{}_2C_1 \cdot {}_3C_1}{{}_5C_2} = \frac{6}{10}$

$P(\text{no green}) = \frac{{}_3C_2}{{}_5C_2} = \frac{3}{10}$

$2/10 + 6/10 - 3/10 = 5/10 = 1/2$

Gain \$0.50

5. A dairy farmer estimates that next year the farm's cows will produce about 25,000 gallons of milk. Because of variation in the market price of milk and the cost of feeding the cows, the profit per gallon may vary with the probabilities given in the table below. Estimate the profit on the 25,000 gallons.

Gain per gallon	$1.10	$0.90	$0.70	$0.40	$0.00	-$0.10
Probability	0.30	0.38	0.20	0.06	0.04	0.02

$\$1.10(.3) + .9(.38) + .7(.2) + .4(.06) + 0(.04) + -.1(.02)$

$.33 + .342 + .14 + .024 + 0 - .002$

$= .834/\text{gallon}$

$25000(.834) = \$20,850$ gained

6. Suppose you play a game in which 5 coins are tossed simultaneously. If 1, 2, 3, or 4 "heads" occur, you win $1 for each "head". If all "heads" or all "tails" occur, however, you lose $20.
   a. Complete the table below:

Number of "heads"	0	1	2	3	4	5
Payoff	-$20	$1	$2	$3	$4	-$20
Probability	1/32	5/32	10/32	10/32	5/32	1/32

   b. What is the game's expected payoff?

$\cancel{-20/32} + 5/32 + \cancel{20/32} + 30/32 + \cancel{20/32} \cancel{- 20/32}$

$= 35/32 \approx \$1.09$