Section 4.1: Venn Diagrams

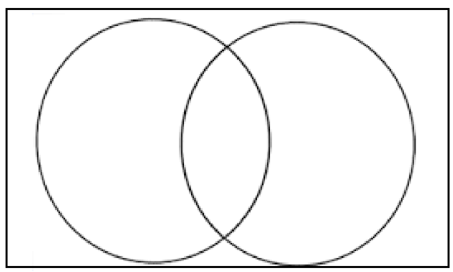
Objectives:

- I can use Venn Diagrams to show intersections and unions.
- I can use Venn Diagrams to solve counting problems.

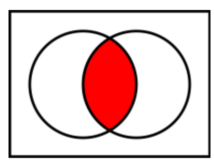
__________________________ is the theory of counting.

When we want to investigate sets of numbers (or objects), we separate the sets of numbers using a **Venn Diagram**.

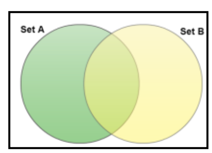
Illustrate the following scenario using the Venn Diagram below:
This diagram represents 1000 typical Americans. Set A contains 410 people who have an allergy to penicillin. Set B contains 100 people who have an allergy to amoxicillin. There are 40 people who have an allergy to both penicillin and amoxicillin.



We notice that sets A and B overlap. When we have an overlap, we call it an _________________ of sets A and B, which is denoted by _______________.



The set of people who have *either* allergy (A or B) represents the __________________________ of sets A and B, which is denoted by ______________.



The set of all elements *not* in set A is called the ___________________ of A and is denoted ____.

In the union diagram above, the unshaded region inside the rectangle represents ______________, which means "the complement of $A \cup B$. What population of people does this region represent?

Look again at our initial Venn Diagram. If $n(A \cup B)$ represents the number of elements in our diagram, what is $n(A \cup B)$? And does $n(A \cup B) = n(A) + n(B)$?

Because the people in the "intersection" are counted twice, we have to compensate for that.

The Inclusion-Exclusion Principle:
For any sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Example 1:
Of the 540 seniors at Meyer High School, 335 are taking math, 287 are taking science, and 220 are taking both math and science. How many students are taking neither math nor science? Represent the situation with a Venn Diagram and show your solution.

Example 2:
In a survey, 113 business executives were asked if they regularly read the *Wall Street Journal*, *Business Week* magazine, and *Time* magazine. The results of the survey are as follows:

88 read the *Journal*.	6 read only the *Journal*.
76 read *Business Week*.	5 read only *Business Week*.
85 read *Time*.	8 read only *Time*.
42 read all three.	

How many executives read none of the three publications? Represent the situation with a Venn Diagram and show your solution.

In this example, we find that ______________________ = ______. Such a set is called an

___________________ and is denoted by the Greek symbol, phi (________).

Section 4.2: Multiplication, Addition, and Complement Principles

Objectives:

- I can use multiplication and addition to solve counting problems.
- I can use the complement principles to solve counting problems.

The Multiplication Principle: If an action can be performed *a* ways, and for each of these ways another action can be performed in *b* ways, then the two actions can be performed together in ________ ways.

Example 1:
If you have 4 sweaters and 2 pairs of jeans, how many different 'sweater and jeans' outfits can you make?

Example 2:
In how many ways can 8 people line up in a cafeteria line?

Notice in this example that no person can occupy more than one space in the line. Thus, the spaces in the cafeteria line are filled ___________________________.

Example 3:
How many license plates can be made using 2 letters followed by 3 digits?

Supposed you could fill each of the spaces with either a letter OR a number. How would that change the number of license plate possibilities?

Note that we __________ the number of ways of performing the actions of choosing a letter and choosing a number because the actions are ______________________________.

The Addition Principle: If two actions are mutually exclusive, and the first can be done in *a* ways and the second in *b* ways, then one action *or* the other can be done in ____________ ways.

Example 4:

In Morse code, the letters of the alphabet are represented by sequences of dots and dashes. For example, a dot and a dash represents the letter A and (dash dash dot) represents letter G. Show that sequences of no more than 4 symbols (dots and dashes) are needed to represent all of the alphabet.

When counting the numbers in a set, sometimes it is easier to count the _________________ of the set.

The Complement Principle: If *A* is a subset of a universal set *U*, then $n(A) = n(U) - n(\overline{A})$.

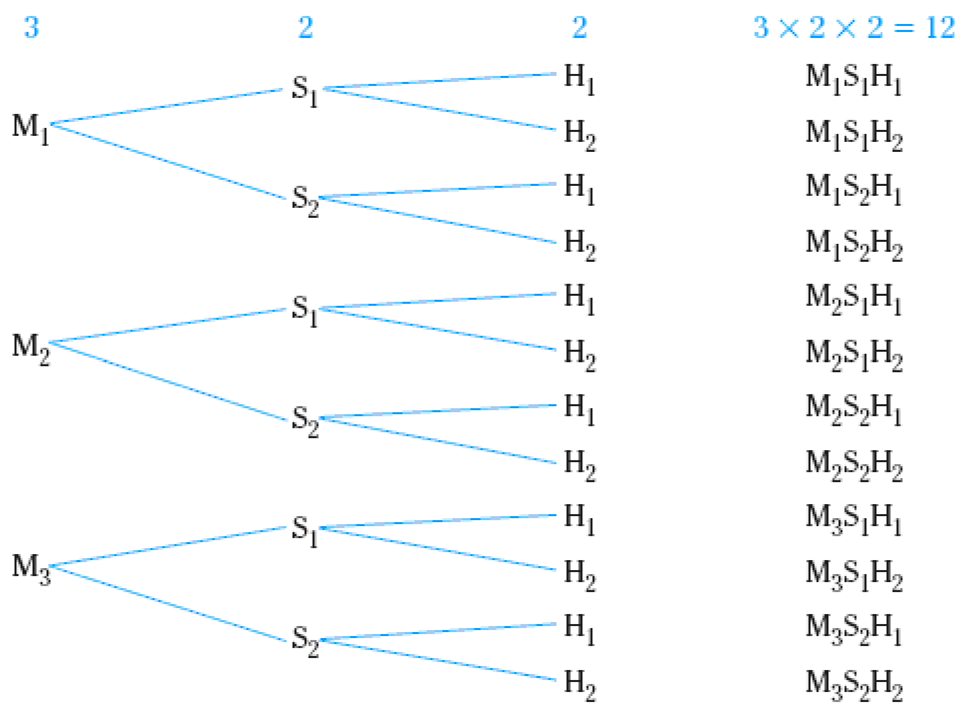
Example 5:

Find the number of 4-digit numbers containing at least one digit 5.

Section 4.3: Permutations and Combinations

Objectives:
- Solve problems related to the basic counting principle.
- Distinguishing between dependent and independent events.
- Solve problems involving permutations or combinations.

Let M represent your math choices, S represent your science choices, and H the humanities courses. Tree Diagrams are used to show all possible choices.



Definition: When one choice does *not* affect the choice of ways to select another choice, the events are called **independent events**.

Definition: Events that do affect each other are called **dependent events.**

Definition: The branch of mathematics that studies different possibilities for the arrangement of objects is called **combinitorics**.

Definition: Suppose one event can be chosen in p different ways, and another independent event can be chosen in q different ways. Then the two events can be chosen successively in pq ways. This is called the **Basic Counting Principle**.

Example 1:

Vickie works for a bookstore. Her manager asked her to arrange a set of five best-sellers for a display. The display is to be set up as shown below. The display set is made up of one book from each of 5 categories. There are 4 nonfiction, 5 science fiction, 3 history, 3 romance, and 3 mystery books from which to choose.

a. Are the choices for each book independent or dependent events?

b. How many different ways can Vickie choose the books for the display?

Definition: The arrangement of objects in a certain order is called a **permutation**. The number of permutations of n objects, taken r at a time is defined as:

Example 2:

During a judging of a horse show at the Fairfield County Fair, there are three favorite horses: Rye Man, Oiler, and Sea of Gus.

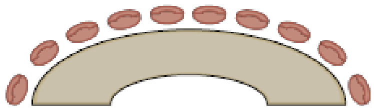
a. Are the selection of first, second, and third place from the three horses independent or dependent events?

b. Assuming there are no ties and the three favorites finish in the top three places, how many ways can the horses win first, second, and third places?

Example 3:

The board of directors of BELA Technology Consultants is composed of 10 members.

a. How many different ways can all the members sit at a conference table as shown?



b. In how many ways can they elect a chairperson, vice-chairperson, treasurer, and secretary, assuming that one person cannot hold more than one office?

Definition: When the order in which an object is selected is *not* a consideration, we would use a **combination**. The number of combinations of *n* objects taken *r* at a time is defined as:

Example 4:

In 1999, The National Art Gallery in Washington, D.C., opened an exhibition of the wors of John Singer Sargent (1856 – 1925). The gallery's curator wanted to select four paintings out of twenty on display to showcase the work of the artist. How many groups of four paintings can be chosen?

Example 5:

At Grant Senior High School, there are 15 names on the ballot for junior class officers. Five will be selected to form a class committee.

a. How many different committees of 5 can be formed?

b. In how many ways can a committee of 5 be formed if each student has a different responsibility?

c. If there are 8 girls and 7 boys on a ballot, how many committees of 2 boys and 3 girls can be formed?

Section 4.4: Permutations with Repetition and Circular Permutations

Objectives:

- Solve problems involving permutations with repetitions.
- Solve problems involving circular permutation.

Complete the table with a **formula**:

	Repetition	No Repetition
Order matters		
Order doesn't matter	--------------------------------	

In the following situations, please identify if repetition is allowed and if order matters.
(**DO NOT SOLVE THE PROBLEMS!**)

1. A bicycle combination lock has four dial wheels with the digits from 0 to 9 on each wheel. How many different lock combinations are possible?

2. How many integers from 100 to 999, inclusive, have three different digits?

3. How many different code words can be constructed from the six letters in the word *string*?

4. There are 13 applicants to fill three sales positions in different departments of a large department store. In how many different ways can these openings be filled?

5. A club with 50 members has the following officers: President, Vice President, Secretary, and Treasurer. If all members of the club are eligible for all offices, how many different slates of officers are possible?

6. How many different four-digit house numbers can be constructed from the six brass numerals 1, 2, 3, 4, 5 and 6?

7. During Halloween, Mrs. Meyer allows trick or treaters to choose 4 pieces of candy from any of 5 bowls set on a table. If there is only one kind of candy in each bowl and no bowl contains the same kind of candy as another bowl, how many different selections are possible?

Definition: The number of permutations of n objects of which p are alike and q are alike is:

Example 1:
How many eight-letter patterns can be formed from the letters of the word *parabola*?

Example 2:
How many eleven-letter patterns can be formed from the letters of the word *Mississippi*?

Definition: If n objects are arranged in a circle, then there are **________________** permutations of the n objects around the circle.

Example 3:
At the Family Friendly Restaurant, nine bowls of food are placed on a circular, revolving tray in the center of the table. You can serve yourself from each of the bowls.

a. Is the arrangement of the bowls on the tray a linear or circular permutation? Explain.

b. How many ways can the bowls be arranged?

Example 4:
Seven people are to be seated at a round table where one person is seated next to a window.

a. Is the arrangement of the people around the table a linear or circular permutation? Explain.

b. How many possible arrangements of people relative to the window are there?

Section 4.5: The Binomial Theorem and Pascal's Triangle

Objectives:

- I can expand a polynomial using Binomial Theorem and Pascal's Triangle.

Expand:

$(x + y)^0$

$(x + y)^1$

$(x + y)^2$

$(x + y)^3$

$(x + y)^4$

$(x + y)^5$

This leads to Pascal's Triangle!!

Example 1:
Use Pascals Triangle to write the first four terms of the expansion of $(x - 2y)^{10}$ in simplified form.

Example 2 (You try it!):

a. Find the first four numbers of the 8^{th} row of Pascal's Triangle.
b. State the first four terms of the expansion of $(x + y)^8$.
c. State the first four terms of the expansion of $(x - y)^8$.