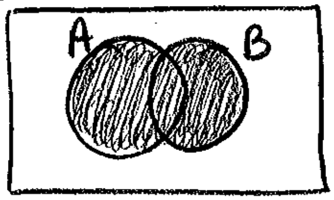
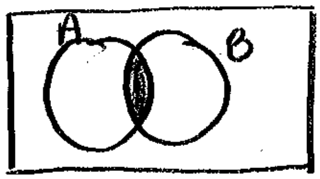
Section 4.1: Venn Diagrams Name Key

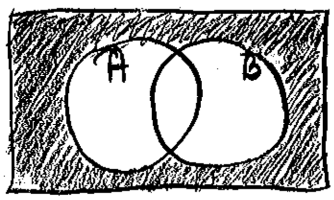
1. Draw separate Venn diagrams for each scenario below. Shade the region representing each set.
   a. $A \cup B$



   b. $A \cap B$



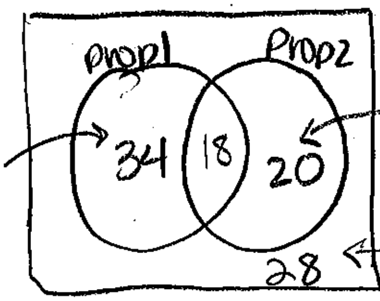
   c. $\overline{A \cup B}$



2. Let U = the universal set of all teachers in your school. Let the subsets of math teachers, biology teachers, physics teachers, and chemistry teachers be represented by M, B, P, and C, respectively. Describe in words each of the following sets, and name a teacher belonging to each set if such a teacher exists in our school.
   a. $M \cup P$ Someone who teaches math or physics or both.
   Mrs. Meyer :)
   b. $M \cap B$ Someone who teaches math and biology.
   Mrs. Lee
   c. $B \cup (P \cap C)$ Someone who teaches biology or physics & chem.
   (or all 3)
   Mr. Hamilton
   d. $\overline{P \cup C}$ Someone who does not teach physics or chem.
   Mrs. Meyer :)
   e. $M \cap \overline{C}$ Someone who teaches math and not chemistry.
   Mrs. Meyer :)

3. Draw a Venn diagram to represent each situation below. Then use the Venn diagram to answer the question.

   a. In an election-day survey of 100 voters leaving the polls, 52 said they voted for Proposition 1, and 38 said they voted for Proposition 2. If 18 said they voted for both, how many voted for neither?



52-18

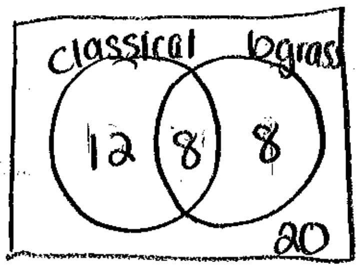
38-18

(34+18+20)

100 - 72

$\boxed{\text{28 people voted for neither}}$

   b. In a survey of 48 high school students, 20 liked classical music and 16 liked bluegrass music. Twenty students said they didn't like either. How many liked classical but not bluegrass?



$n(c) + n(b) - n(c \cap b) = 28$

$20 + 16 - x = 28$

$36 - x = 28$

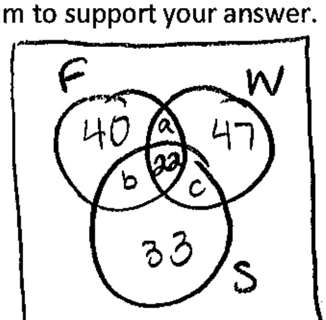
$-x = -8$

$x = 8$

4. Of the 415 girls at Gorham High School last year, 100 played fall sports, 98 played winter sports, and 96 played spring sports. Twenty-two girls played sports all three seasons, while 40 played only in the fall, 47 only in winter, and 33 only in spring.

   a. How many girls played fall and winter sports, but not a spring sport? Use a Venn diagram to support your answer.

415 =



F: $100 = 40 + 22 + a + b$

W: $98 = 47 + 22 + a + c$

S: $96 = 33 + 22 + b + c$

$38 = a + b$

$29 = a + c$ $\quad a = 29 - c$

$41 = b + c$ $\quad b = 41 - c$

$38 = 29 - c + 41 - c$

$38 = 70 - 2c$ $\quad c = 16 \quad b = 25 \quad \boxed{a = 13}$

   b. How many girls did not play sports in any of the three seasons?

$F \cup W \cup S = 40 + 47 + 33 + 22 + 16 + 25 + 13 = 196$ athletes

$\overline{F \cup W \cup S} = 415 - 196 = 219$ girls did not play sports

Section 4.2: Multiplication, Addition, and Complement Principles

1. Evaluate:
    a. 2! = 2
    b. 5! = 120
    c. 4! = 24

2. A girl has 6 different skirts and 10 different blouses.
    a. How many different skirt-blouse outfits are possible?

       $6 \cdot 10 = 60$ outfits

    b. If she also has 3 different sweaters, how many skirt-blouse-sweater outfits are possible?

       $3 \cdot 6 \cdot 10 = 180$ outfits

3. If 10 runners compete in a race, in how many different ways can prizes be awarded for first, second, and third places?

   $\underline{10} \cdot \underline{9} \cdot \underline{8} = 720$ ways

4. How many different ways can you answer 10 true-false questions?

   $2^{10} = 1024$ ways

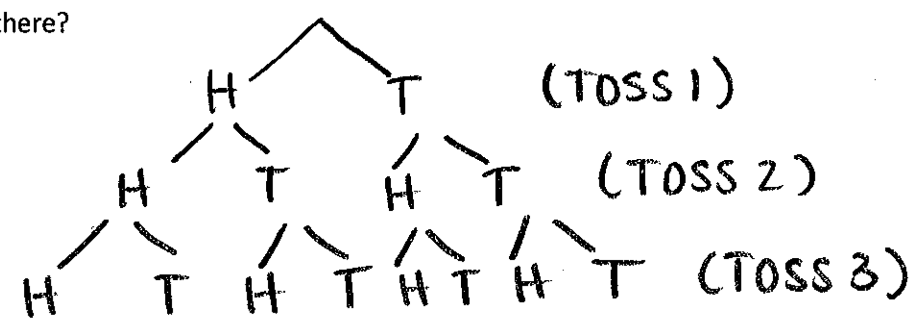
5. How many 3-digit numbers can be formed using the digits 4, 5, 6, 7, and 8 if:
    a. The digits can be repeated?

       $5^3 = 125$ ways

    b. The digits cannot be repeated?

       $5 \cdot 4 \cdot 3 = 60$ ways

6. Make a tree diagram showing the outcomes if a coin is tossed 3 times. How many possible outcomes are there?



8 outcomes

#If letters/#'s can be in any order...

then $35^5 + 35^6 = 1,890,787,500$

7. How many possibilities are there for a license plate with 2 letters and 3 or 4 nonzero digits? $26+9=35$

$\underline{26}\ \underline{26}\ \underline{9}\ \underline{9}\ \underline{9} + \underline{26}\ \underline{26}\ \underline{9}\ \underline{9}\ \underline{9}\ \underline{9}$

$676 \cdot 729 \qquad 676 \cdot 6561$

$492,804 + 4435236 = 4,928,040$

8. How many numbers from 5000 to 6999 contain at least one 3?

U = all possible

A = at least one 3

$\bar{A}$ = no 3's

$n(U) - n(\bar{A})$

$\underline{2}\ \underline{10}\ \underline{10}\ \underline{10} - \underline{2}\ \underline{9}\ \underline{9}\ \underline{9}$

$2000 - 1458 = 542$

9. Consider the scenario of choosing a 3-digit number:
   a. How many 3-digit numbers contain no 7's?

      $\underline{8}\ \underline{9}\ \underline{9} = 648$

      not 0 or 7

   b. How many 3-digit numbers contain at least one 7?

      All 3 digit − no 7's

      $\underline{9}\ \underline{10}\ \underline{10} - 648 = 252$

10. A school has 677 students. Explain why at least two students must have the same pair of initials (first and last).

$\underline{26} \cdot \underline{26} = 676$ possibilities...

so with 677 students, someone would share initials!

Section 4.3: Permutations and Combinations

1. If you toss a coin, then roll a die, and then spin a 4-colored spinner with equal sections, how many outcomes are possible?

   $2 \cdot 6 \cdot 4 = 48$ outcomes

2. How many ways can 7 classes be scheduled, if each class is offered in each of 7 periods?

   $_7P_7 = \frac{7!}{(7-7)!} = 7! = 5040$

3. Find the number of different 7-digit telephone numbers where:
   a. The first digit cannot be zero. $9 \cdot 10^6 = 9{,}000{,}000$
   b. Only even digits are used.
   $\underline{4}\ \underline{5}\ \underline{5}\ \underline{5}\ \underline{5}\ \underline{5}\ \underline{5} = 4 \cdot 5^6 = 62{,}500$
   can't be 0 or odd
   c. The complete telephone numbers are multiples of 10. $10^6 \cdot 1 = 1{,}000{,}000$
   $\underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10} \cdot \underline{1}$ must end in 0
   d. The first three digits are 593 in that order.
   $\underline{1}\ \underline{1}\ \underline{1}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = 1^3 \cdot 10^4 = 10{,}000$

4. State whether the following events are *dependent* or *independent*.
   a. Selecting members for a team. dependent
   b. Tossing a penny, rolling a die, and then tossing a dime. independent
   c. Deciding the order in which to complete your homework assignments. dependent

5. Find each value (show the work!!)
   a. $P(8, 8)$ $\frac{8!}{(8-8)!} = 8! = 40{,}320$
   b. $P(5, 3)$ $\frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!}} = 60$
   c. $P(9, 5)$ $\frac{9!}{(9-5)!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = 15{,}120$

6. Find each value (show the work!!)
   a. $\frac{P(6,3)}{P(4,2)} = \frac{120}{12} = 10$
   b. $\frac{P(6,3) \cdot P(7,5)}{P(9,6)}$ $\frac{302400}{60480} = 5$

7. Find each value (show the work!!)
   a. $C(10, 5)$ $\frac{10!}{(10-5)!5!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot \cancel{5!}}{5!\,\cancel{5!}} = \frac{30240}{120} = 252$
   b. $C(12,4)$ $\frac{12!}{(12-4)!4!} = \frac{12\cdot 11\cdot 10\cdot 9\cdot \cancel{8!}}{\cancel{8!}\,4!} = \frac{11880}{24} = 495$
   c. $C(7,3)\cdot C(8,5)$ $= 3432$

8. A pizza shop has 14 different toppings from which to choose. How many different 4-topping pizzas can be made?

$_{14}C_4 = 1001$

9. How many different 12 member juries can be formed from a group of 18 people?

$_{18}C_{12} = 18{,}564$

10. How many different ways can 11 paintings be displayed on a wall?

$_{11}P_{11} = 39{,}916{,}800$

11. From a standard 52-card deck, find how many 5-card hands are possible that have:
   a. 3 hearts and 2 clubs? $_{13}C_3 \cdot {_{13}C_2} = 22{,}308$
   b. 1 ace, 2 jacks, and 2 kings? $_4C_1 \cdot {_4C_2} \cdot {_4C_2} = 144$
   c. All face cards? $_{12}C_5 = 792$

12. A home security company offers a security system that uses the numbers 0 through 9, inclusive, for a 5-digit security code.
   a. How many different security codes are possible?
      $10^5 = 100{,}000$
   b. If no digits can be repeated, how many security codes are available?
      $10\cdot 9\cdot 8\cdot 7\cdot 6 = 30{,}240$
   c. Suppose the homeowner does not want to use 0 as one of the digits and wants only two of the digits to be odd. How many codes can be formed if the digits can be repeated?
      $5^2\cdot 4^3 = 1600$
   d. Using the same scenario as part (c), if no repetitions are allowed, then how many codes can be formed?
      $_5P_2 \cdot {_4P_3} = 480$

Section 4.4: Permutations with Repetitions and Circular Permutations

1. How many different ways can the letters of each word be arranged?
   a. *Pizzeria* $\frac{8!}{2!2!} = 10{,}080$
   2-z; 2-i
   b. *California* $\frac{10!}{2!2!} = 907{,}200$
   c. *Calendar* $\frac{8!}{2!} = 20{,}160$
   d. *Trigonometry* $\frac{12!}{2!2!2!} = 59{,}875{,}200$

2. How many different 7-digit phone numbers can have the digits 7, 3, 5, 2, 7, 3, and 2.
   $$\frac{7!}{2!2!2!} = 630$$

3. Five country posters and four rap posters are to be placed in a display window. How many ways can they be arranged if they are placed by category? $\frac{9!}{5!4!} = 126$

4. Determine whether each arrangement of objects is a *linear* or *circular* permutation. Then determine the number of arrangements for each situation.
   a. 12 gondolas on a Ferris wheel.
   circular; $(12-1)! = 39{,}916{,}800$
   b. A stack of 6 pennies, 3 nickels, 7 dimes, and 10 quarters.
   linear; $\frac{26!}{6!3!7!10!} = 5.1\times10^{12} = 5{,}109{,}000{,}000{,}000$
   c. The placement of 9 specialty departments along the outside perimeter of a market.
   circular; $(9-1)! = 40{,}320$
   d. A family of 5 seated around a rectangular table.
   circular; $(5-1)! = 24$
   e. 8 tools on a utility belt.
   circular; $(8-1)! = 5040$

5. To break a code, Zach needs to find how many symbols there are in a particular sequence. He is told that there are 3 *x*'s and some dashes. He is also told that there are 35 linear permutations of the symbols. What is the total number of symbols in the code sequence?

$3! \cdot 35 = \frac{x!}{3!(x-3)!} \cdot 3!$

$210 = \frac{x\cdot(x-1)\cdot(x-2)\cdot\cancel{(x-3)!}}{\cancel{(x-3)!}}$

$x(x-1)(x-2) = 210$
Solve on calc.
$x = 7$
$7\cdot6\cdot5 = 210$

Section 4.5: The Binomial Theorem and Pascal's Triangle

1. Give the expansion of each binomial below. Simplify your answers.
   a. $(x + y)^4$ $1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

   b. $(10 + 1)^4$ $(10)^4 + 4(10)^3(1) + 6(10)^2(1)^2 + 4(10)(1)^3 + 1^4$
   $= 14641$

   c. $(10 - 1)^4$ $(10)^4 - 4(10)^3(1) + 6(10)^2(1)^2 - 4(10)(1)^3 + 1^4$
   $= 6561$

2. Find the first four terms in the expansion of $(a^2 - b)^{100}$. You do NOT need to simplify.

$${}_{100}C_0 (a^2)^{100}(-b)^0 + {}_{100}C_1 (a^2)^{99}(-b)^1 + {}_{100}C_2 (a^2)^{98}(-b)^2 + {}_{100}C_3 (a^2)^{97}(-b)^3$$

$$1a^{200} - 100a^{198}b + 4950a^{196}b^2 - 161700a^{194}b^3$$

3. In the expansion of $(x + y)^{12}$:
   a. What is the coefficient of x^8y^4?

   $${}_{12}C_4\, x^8y^4 = 495x^8y^4$$

   b. What is the coefficient of x^4y^8?

   $${}_{12}C_8\, x^4y^8 = 495x^4y^8$$