Unit 3 Homework: Graphs and Circuits Name___________________________

Section 3.1: Modeling with Graphs

In #1 – 4, suppose that a reporter conducts a poll of voters regarding a plan to construct a new sports stadium using city funds. Of those polled, 77% are people who vote regularly, while 23% vote only occasionally. Of the regular voters, 42% are in favor of the plan to construct the stadium. Among the occasional voters, 54% favor its construction.

1. Draw a probability tree to represent this situation.

2. If 1,500 voters respond, how many are regular voters not in favor of the plan? ______________

3. Which category contains the largest percentage of respondents? ______________

4. What is the probability that a randomly selected voter who is not in favor of the plan is a regular voter?

5. Suppose the process of conducting an archaeological dig involves the following tasks:

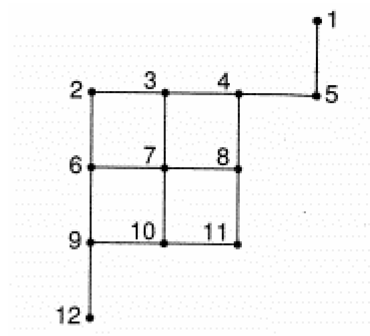
Task	Description	Time Required (Days)	Prerequisite Tasks
A	choose a site	1	none
B	organize a team	3	none
C	travel to site	1	A, B
D	set up equipment	2	C
E	dig	8	D
F	keep journal of findings	9	D
G	write a report of findings	12	E, F

a) Sketch a directed graph to represent this situation.

b) What is the minimal time required to conduct the entire dig?

In #6 – 7, the vertices of the graph at the right represent traffic lights and the edges represent streets.

6. Draw a path that will allow a maintenance person to pass each light exactly once.



7. Suppose light 8 and the part of the street connecting it to light 7 are removed (so lights 4 and 11 are connected directly). Would a path satisfying #6 still be possible?

Section 3.2: Definition of a Graph Name_________________________

In #1 – 3, draw a graph with the specified characteristics.

1. Two vertices and three edges.

2. An isolated vertex, two loops, and three edges.

3. Two parallel edges, three vertices, and a loop.

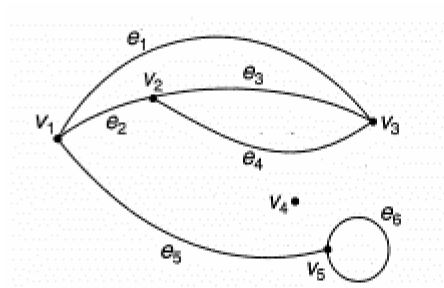
4. Draw the graph *G* defined as follows:

set of vertices: $\{v_1, v_2, v_3, v_4, v_5\}$;
set of edges: $\{e_1, e_2, e_3, e_4\}$

edge-endpoint function:

edge	endpoints
e_1	$\{v_1\}$
e_2	$\{v_2, v_4\}$
e_3	$\{v_3, v_5\}$
e_4	$\{v_1, v_3\}$

5. Use the graph at the right:



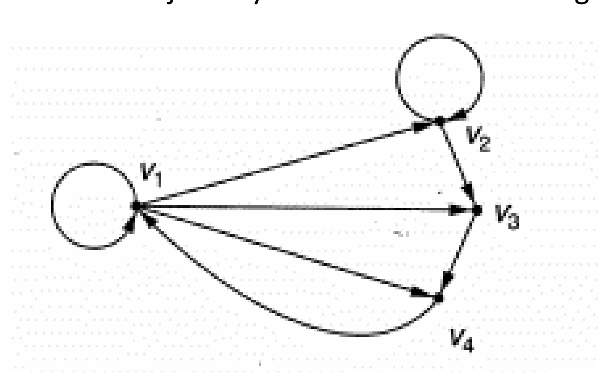
   a) Identify any isolated vertices. ________________

   b) Identify all edges adjacent to edge 3. ________________

   c) Identify any loops. ______________

   d) Give the edge-endpoint function table for the graph.

6. Write the adjacency matrix for the directed graph below.



7. What does the sum of the components of the matrix for a directed graph represent?

8. Draw an undirected graph with the following adjacency matrix.

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \end{array}$$

Section 3.3: The Handshake Problem Name___________________________

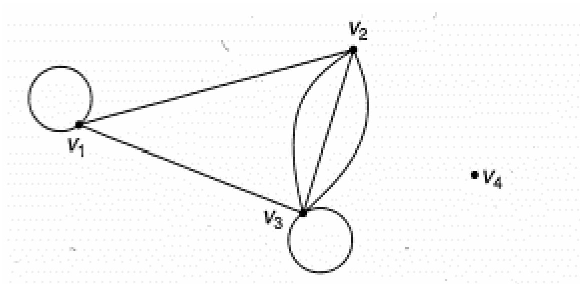
1. Draw a graph with four vertices of the following degrees: 1, 3, 3, and 5.

2. Draw a simple graph with three vertices of degrees 1, 2, and 1.

3. Fill in the blanks with the word "even" or "odd". "*Every graph has an* ________________ *number of vertices of* _________________ *degree*."

4. A graph has four edges. What is its total degree? __________________________

5. Refer to the graph below.
   a. Give the degree of each vertex:_________________________________
   b. Give the total degree of the graph. ______________________________



In 6 and 7, either draw a graph with the given properties or explain why no such graph exists.

6. A graph with five vertices of degrees 1, 2, 2, 3, and 5.

7. A graph with six vertices of degrees 1, 1, 2, 3, 4, and 5.

8. 33 parents meet at a graduation ceremony. Is it possible for each parent to shake hands with exactly seven other parents? Explain.

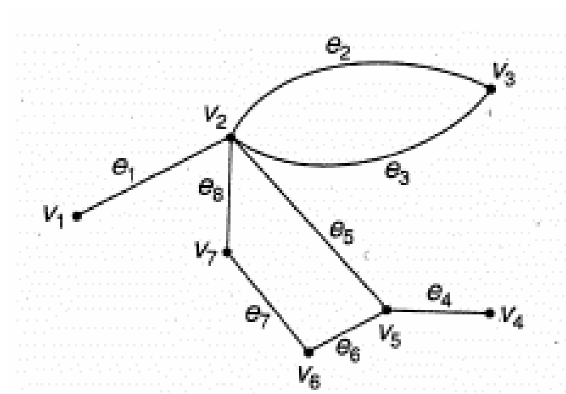
9. The instructor of a Spanish class requests that each one of the 12 students in the class meet with three other students, one at a time, to practice speaking Spanish.
   a. Draw a graph to represent the situation.

   b. How many different pairings result? ________________________________

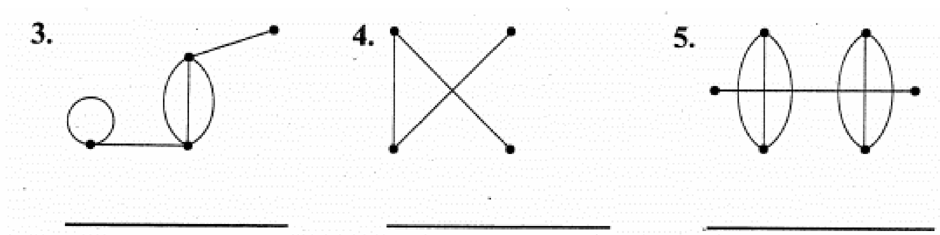
Section 3.4: Konigsberg Bridge Problem Name____________________________

1. Draw a graph with 4 vertices, one loop, and 2 parallel edges, and which has an Euler circuit. Identify the circuit.

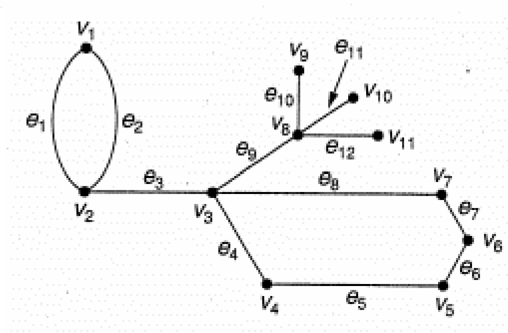
2. Consider the graph at the right. Describe each of the following, if possible:
   a. A walk from v_1 to v_4.
   b. A path from v_4 to v_7.
   c. An Euler circuit starting at v_1.
   d. A walk from v_1 to v_4 that is not a path.



In #3 – 5, determine whether or not the graph is connected.



6. Consider the graph at the right.
   a. Give each edge whose removal would keep the graph connected.
   b. What is the maximum number of edges that can be removed simultaneously while keeping the graph connected?



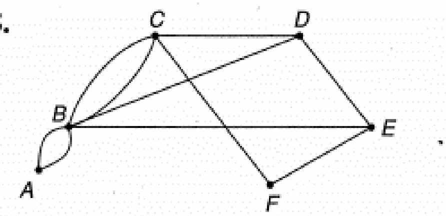
7. Consider the statement: *If every vertex of a graph has even degree, then the graph has an Euler circuit.*"
    a. Write the converse of that statement.

    b. Which is true, the statement or the converse?

    c. What additional characteristic, if possessed by the graph, makes both statements true?

In #8 – 9 determine whether or not the graph described has an Euler circuit. Justify your answer.

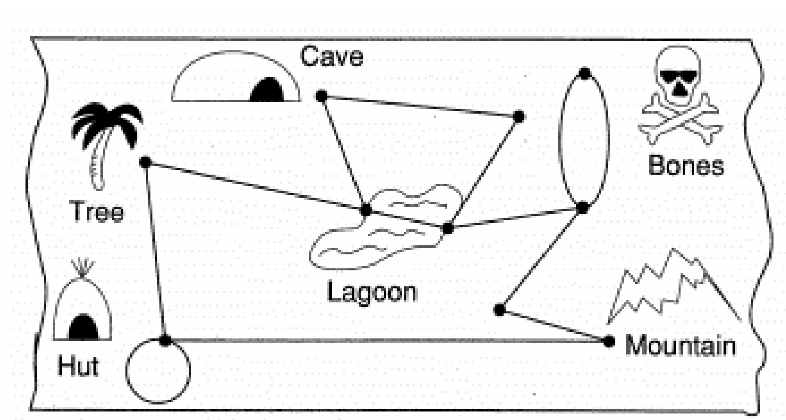
8.



9. the graph with adjacency matrix

$$\begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 2 & 0 & 0 & 4 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 4 & 0 & 0 & 2 \end{bmatrix} \end{array}$$

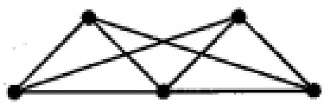
10. Suppose a pirate finds a treasure map like the one below. If the treasure is at one end of the points marked, is it possible for the pirate to search at every point, starting and ending at the same point and using each edge only once? If so, draw arrows on the map to show a possible route. If not, draw arrows on the map to show a route which starts and ends at the same point and passes through as many other points as possible.



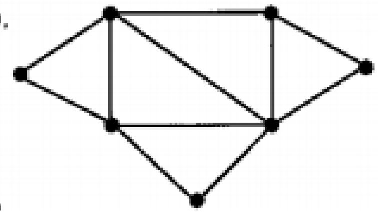
Section 3.5: Hamiltonian Circuits and Paths and Graph Coloring

1. Which of the graphs have Hamiltonian circuits? If it has one, label it.

a.

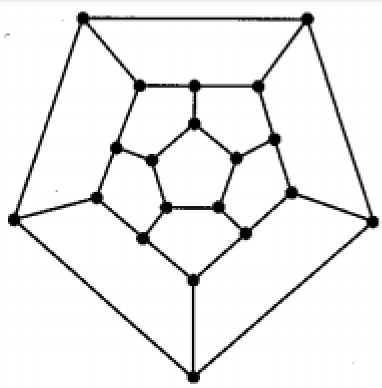


b.



2. Give one example (not already mentioned) of a situation that could be modeled by a graph in which finding a Hamiltonian path or circuit would be a benefit.

3. Hamiltonian's Icosian game was played on a wooden regular dodecahedron (a solid with 12 sides). Here is a planar representation of the game:



   a. Find a Hamiltonian circuit for the graph.

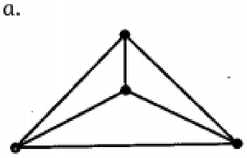
   b. Is there only one Hamiltonian circuit for this graph?

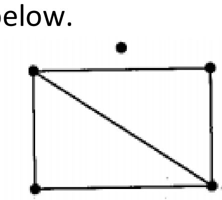
   c. Can the circuit begin at any of the vertices? Or only at some of them?

4. Draw a tournament with five players, in which player A defeats everyone, B defeats everyone but A, C is defeated by everyone, and D defeats E.

5. Find the chromatic number for each of the graphs below.

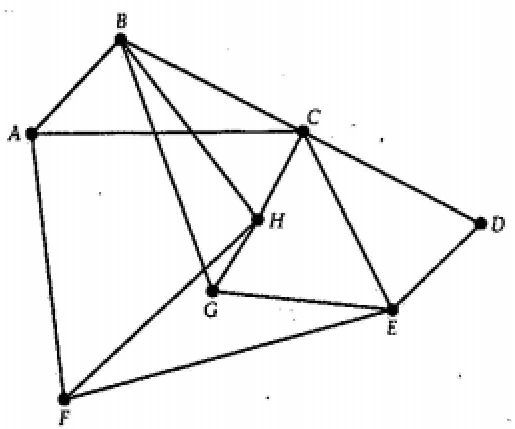
a.



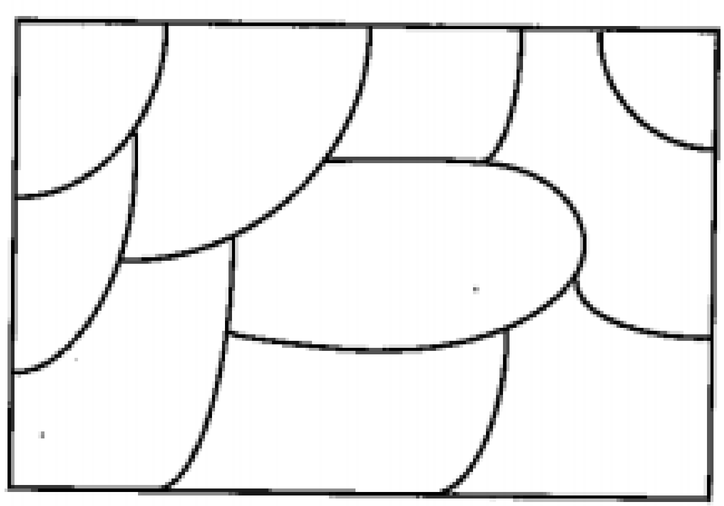


6. If a graph has a chromatic number of 1, what do you know about the graph?

7. Ms. Suzuki is planning to take her history class to the art museum. Following is a graph showing those students who are not compatible. Assuming that the seating capacity of the cars is not a problem, what is the minimum number of cars necessary to take the students to the museum?

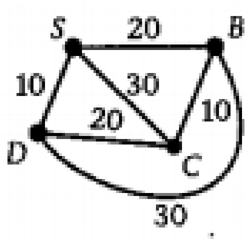


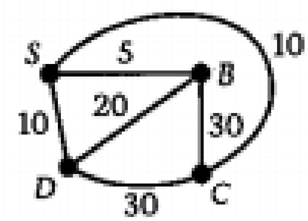
8. Color the following map using only 3 colors.



Section 3.6: The Traveling Salesperson Problem/The Shortest Path

1. For each graph below, complete the following:
    a. Construct a tree diagram showing all possible circuits that begin at S, visit each vertex of the graph exactly once, and end at S.
    b. Find the total weight of each route.
    c. Identify the shortest route.



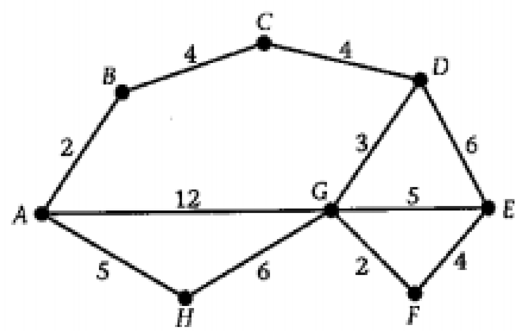


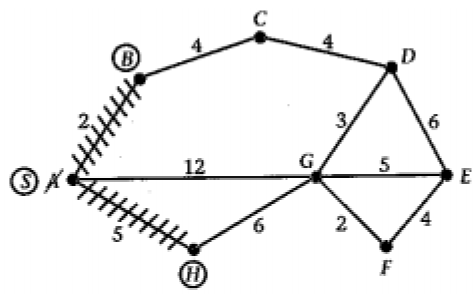
2. A delivery person must visit each of his warehouses daily. His delivery route begins and ends at his garage (G). The table below shows the approximate travel time (in minutes) between stops.

	G	A	B	C	D
G	--	25.0	16.5	18.0	43.0
A	25.0	--	21.0	17.5	15.0
B	16.5	21.0	--	23.5	19.5
C	18.0	17.5	23.5	--	21.0
D	43.0	15.0	19.5	21.0	--

    a. Draw a weighted complete graph with 5 vertices to represent the information in the table.
    b. Find the shortest route.
    c. What is the travel time for the shortest route?

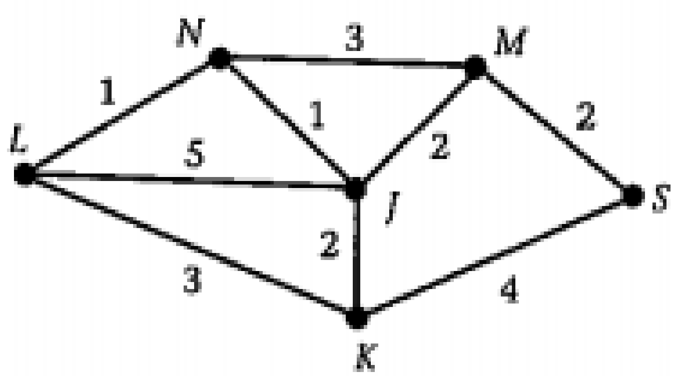
3. Julian began using the shortest path algorithm to find the shortest route from A to E for the graph below. The work that he was able to complete before he had to stop is shown. Fill in the missing distances, vertex, and edge in step 3. Then complete Julian's problem of using the shortest path algorithm to find the shortest path from A to E.





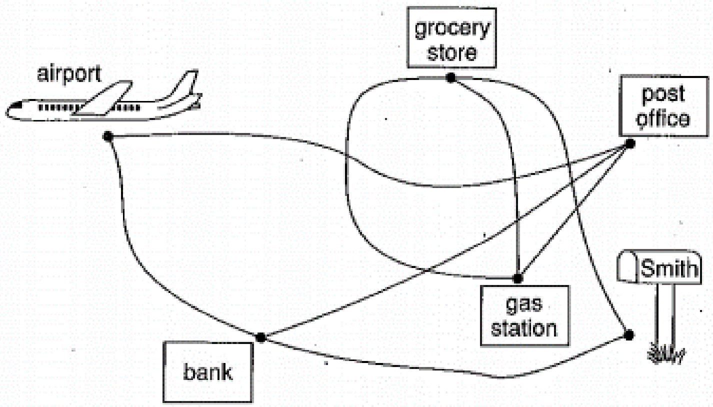
1. SB – 2
   SC – 5 Circle *B*, darken *SB*.
   SG – 12
2. SBC – 6
   SG – 12 Circle *H*, darken *SH*.
   SH – 5
3. SBC – ?
   SG – ? Circle ?, darken ?
   SHG – ?

4. Use the shortest path algorithm to find the shortest distance from S to each of the other vertices in the following graph.



Unit 3 Review Name____________________

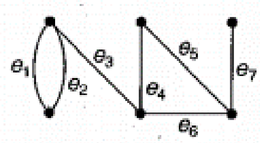
1. A map is shown below. Mr. Smith is going to the airport to pick up his wife. Can he leave home, pick up his wife, and return home, stopping at the bank, the gas station, the grocery store, and the post office on the way, without passing any location more than once? Justify your answer. **1.** ____________________



2. Draw a directed graph whose adjacency matrix is given below. **2.**

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

3. Consider the graph at the right. Identify an edge such that when the edge is removed, the graph is



a. simple. **3. a.** ____________________

b. not connected. **b.** ____________________

4. Draw a simple graph with five vertices of degrees 2, 3, 3, 4, and 4. **4.**

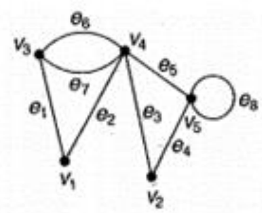
5. Does the complete graph of seven vertices have an Euler circuit? Justify your answer. **5.** ____________________

6. If the matrix below is the adjacency matrix of an undirected graph, find the values of x and y. **6.** ________

$$\begin{bmatrix} 0 & 4 & 1 & 2 \\ 4 & 1 & 0 & 3 \\ 1 & 0 & 4 & y \\ x & 3 & 0 & 2 \end{bmatrix}$$

7. Draw a simple graph with seven vertices of degrees 1, 1, 2, 3, 4, 5, and 5, or show that no such graph exists.

10. Use the graph shown at the right.



a. Identify all vertices adjacent to v_4. **10. a.** ________

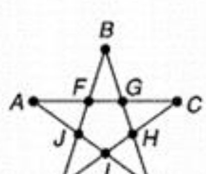
b. Identify any parallel edges. **b.** ________

c. The removal of which two edges would create a simple graph? **c.** ________

11. a. Does a simple, connected graph with vertices of degrees 2, 2, 2, 3, and 3 contain an Euler circuit? **11. a.** ________

b. Justify your answer. **b.** ________

12. a. Can the figure at the right be drawn without lifting one's pencil? If so, show how. **12. a.** ________



b. Does it contain an Euler circuit? Justify your answer. **b.** ________

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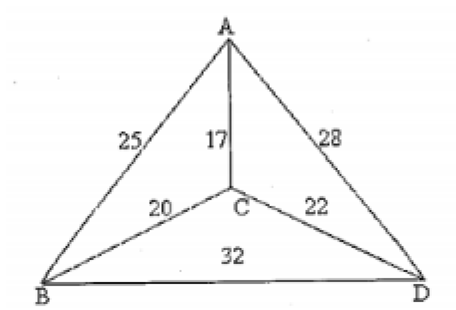
13. Use the graph below to find the lowest cost for a traveling salesman starting at point C, visiting each location, and finishing his trip back at point C. Prove your answer using a tree diagram.



14. Sunshine Child Care Center is organizing classes at the local YMCA for the 5-year-olds. The available classes and the children who want to take each class are given in the chart below.

	Swimming	Tae Kwan Do	Gymnastics	T-Ball	Soccer
Zack	X	X			X
Sydney			X		X
Sam		X		X	
Max	X		X		X
Anne	X		X		
Van		X			X
Todd	X	X			X
Janyce	X		X	X	
Mark		X			X

a. How many different time periods will be needed to hold these classes?
b. At the last minute, Anne decides to take Tae Kwan Do also. Does this change the number of time periods needed to hold classes?