

Unit 2: Matrices – TEST REVIEW

Use the following matrices for #1 – 2.

$$A = \begin{bmatrix} 2 & 0 \\ 4 & 7 \\ -1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -1 & 2 \\ 1 & 6 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 4 & -2 \\ -1 & 0 \\ 3 & -4 \end{bmatrix} \quad D = \begin{bmatrix} 1 & -3 \\ 2 & -2 \\ 3 & -1 \end{bmatrix} \quad E = \begin{bmatrix} -5 & 0 & -3 \\ 1 & 2 & 9 \\ -8 & 7 & 6 \end{bmatrix}$$

1. What is the value of C_{12} ? -2
2. Find the value of each of the given expressions. If it is not possible, state why.

a. $A + C$ $\begin{bmatrix} 6 & -2 \\ 3 & 7 \\ 2 & -1 \end{bmatrix}$

e. BA $\begin{bmatrix} 2 & -1 \\ 29 & 33 \end{bmatrix}$

b. $C - B$ Not possible...
must be same dimensions

f. AB $\begin{bmatrix} 8 & -2 & 4 \\ 23 & 38 & -13 \\ -1 & 19 & -11 \end{bmatrix}$

c. $(A + C) - D$ $\begin{bmatrix} 5 & 1 \\ 1 & 9 \\ -1 & 0 \end{bmatrix}$

g. $DB - E$ $\begin{bmatrix} 6 & -19 & 14 \\ 5 & -16 & 1 \\ 19 & -16 & 3 \end{bmatrix}$

d. $2A + D$ $\begin{bmatrix} 5 & -3 \\ 10 & 12 \\ 1 & 5 \end{bmatrix}$

h. C^T $\begin{bmatrix} 4 & -1 & 3 \\ -2 & 0 & -4 \end{bmatrix}$

3. Three music classes at Central High are selling candy as a fundraiser. The number of each kind of candy sold by each of the three classes is shown in the following table.

	Jazz Band	Symphonic Band	Orchestra
Almond Bars	300	220	250
Chocolate Chews	240	330	400
Mint Patties	150	200	180
Sour Gummies	175	150	160

The profit for each type of candy is: sour gummies, 30 cents; chocolate chews, 50 cents; almond bars, 25 cents; and mint patties, 35 cents. Calculate the profit made by each class on its candy sales.

$$\begin{bmatrix} AB & CC & MP & SG \\ .25 & .50 & .35 & .30 \end{bmatrix} \begin{bmatrix} JB & SB & Orch \\ 300 & 220 & 250 \\ 240 & 330 & 400 \\ 150 & 200 & 180 \\ 175 & 150 & 160 \end{bmatrix} = \begin{bmatrix} JB & SB & Orch \\ \$300 & \$335 & \$373.50 \end{bmatrix}$$

4. Mr. Jones has been shopping for a vacuum-powered cleaning system. He found one at Z-Mart and another model at Base Hardware. The Z-Mart system cost \$39.50, disposal cartridges were 6 for \$24.50, and storage cases were \$8.50 each. At Base Hardware, the system cost \$49.90, cartridges were 6 for \$29.95, and cases were \$12.50 each.

- a. Write and label a matrix showing the prices for the three items at the two stores.

Label your rows and columns of your matrix.

$$A = \begin{matrix} & \text{system} & \text{cart.} & \text{case} \\ \text{ZM} & [\$39.50 & 24.50 & 8.50 \\ \text{BH} & [49.90 & 29.95 & 12.50 \end{matrix}$$

- b. Mr. Jones decided to wait and see if the prices for the systems would be reduced during the upcoming sales. When he went back during the sales, the Z-Mart prices were reduced by 10% and the Base Hardware prices were reduced by 20%. Construct a new matrix showing the sale prices for each of the three items at the two stores. Label your rows and columns of your matrix.

$$B = \begin{matrix} & \text{syst.} & \text{cart.} & \text{case} \\ \text{ZM} & [\$35.55 & 22.05 & 7.65 \\ \text{BH} & [39.92 & 23.96 & 10 \end{matrix}$$

- c. Using a matrix, show how much Mr. Jones would save for each item at the two stores. Label your rows and columns of your matrix.

$$A - B = \begin{matrix} & \text{Syst} & \text{cart} & \text{case} \\ \text{ZM} & [3.95 & 2.45 & .85 \\ \text{BH} & [9.98 & 5.99 & 2.50 \end{matrix}$$

- d. Suppose Mr. Jones is interested in purchasing the systems for himself and 3 of his friends. How much would he have to pay for each of the 3 items at the two stores if he purchased them at the sale prices? Label your rows and columns of your matrix.

$$3B = \begin{matrix} & \text{Syst} & \text{cart} & \text{case} \\ \text{ZM} & [106.65 & 66.15 & 22.95 \\ \text{BH} & [119.76 & 71.88 & 30 \end{matrix}$$

=4

$$4B = \begin{matrix} & \text{Sys.} & \text{Cart.} & \text{Case} \\ \text{Z} & [\$142.20 & \$88.20 & \$30.60 \\ \text{B} & [\$159.68 & \$95.84 & \$40 \end{matrix}$$

5. An artist creates plates and bowls from small pieces of colored woods. She currently has orders for five plates, three large bowls, and seven small bowls. Each plate requires 100 pieces of ebony, 800 pieces of walnut, 600 pieces of rosewood, and 400 pieces of maple. It takes 200 ebony pieces, 1200 walnut pieces, 1000 rosewood pieces and 800 pieces of maple to make a large bowl. A small bowl takes 50 pieces of ebony, 500 pieces of walnut, 450 pieces of rosewood, and 400 pieces of maple.

a. Write a row matrix showing the current orders for this artist's work. Use labels!

oops!

$$W = \begin{matrix} & \begin{matrix} \text{Eb} & \text{WN} & \text{RW} & \text{MP} \end{matrix} \\ \begin{matrix} P \\ LB \\ SB \end{matrix} & \begin{bmatrix} 100 & 800 & 600 & 400 \\ 200 & 1200 & 1000 & 800 \\ 50 & 500 & 450 & 400 \end{bmatrix} \end{matrix}$$

b. Write a matrix showing the number of pieces of wood used in an individual plate or bowl. Label the rows and columns of your matrix.

$$\text{Order} = \begin{matrix} & \begin{matrix} P & LB & SB \end{matrix} \\ \begin{bmatrix} 5 \\ 3 \\ 7 \end{bmatrix} & \end{matrix}$$

c. Compute the number of pieces of each type of wood the artist will need for the plates and bowls that are on order.

$$\text{Order} \times W = \begin{matrix} & \begin{matrix} \text{Eb} & \text{WN} & \text{RW} & \text{MP} \end{matrix} \\ \begin{bmatrix} 1450 & 11100 & 9150 & 7200 \end{bmatrix} & \end{matrix}$$

d. Suppose it takes the artist 3 weeks to fashion a plate, 4 weeks to make a large bowl, and 2 weeks to complete a small bowl. How long will it take the artist to fill all the orders for the plates and bowls? Show matrix work.

$$\begin{matrix} \begin{bmatrix} P & LB & SB \\ 5 & 3 & 7 \end{bmatrix} & \begin{matrix} P \\ LB \\ SB \end{matrix} \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix} & = & [41 \text{ weeks}] \end{matrix}$$

6. Are the following matrices are inverses of each other? Explain your answer.

$$\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix} \text{ and } \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$$

$[A] \qquad [B]$

$$A \times B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$B \times A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Yes, A and B are inverses

7. Find the inverse of the following matrices, or explain why no inverse exists.

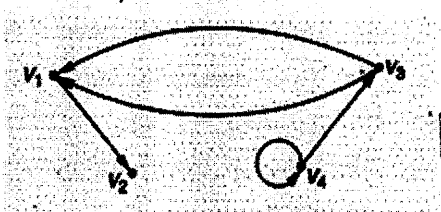
$$\begin{bmatrix} -1 & 3 \\ 2 & -6 \end{bmatrix} \det = 6 - 6 = 0$$

inverse cannot be found because the determinant is 0!

$$\begin{bmatrix} 4 & -2 \\ 7 & -5 \end{bmatrix} \det = -20 - (-14) = -6$$

$$-\frac{1}{6} \begin{bmatrix} -5 & 2 \\ -7 & 4 \end{bmatrix} = \begin{bmatrix} 5/6 & -1/3 \\ 7/6 & -2/3 \end{bmatrix}$$

8. Write a matrix that represents the communication network below.



$$M = \begin{matrix} & \begin{matrix} v_1 & v_2 & v_3 & v_4 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{matrix}$$

9. Using the matrix in #8, how many ways are there to relay a message from v4 to v1 using exactly one relay (2 steps)? Show work leading to your answer.

$$M^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix}$$

2 ways
(row 4, col 1)

10. The characteristics of the female population of a herd of small mammals are shown in the following table.

$$L = \begin{bmatrix} 0 & 0.6 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0.8 & 0 & 0 & 0 \\ 1.1 & 0 & 0 & 0.9 & 0 & 0 \\ 0.9 & 0 & 0 & 0 & 0.8 & 0 \\ 0.4 & 0 & 0 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Age (months)	Birth Rate	Survival Rate
0-4	0	0.6
4-8	0.5	0.8
8-12	1.1	0.9
12-16	0.9	0.8
16-20	0.4	0.6
20-24	0	0

Suppose the initial female population for the herd is given by $P_0 = [22 \ 22 \ 18 \ 20 \ 7 \ 2]$

a. What is the expected lifespan of the animal? 24 months

b. How many newborns will there be after 4 months (1 cycle)?

$$P_0 \cdot L = \begin{bmatrix} 0-4 & 4-8 & 8-12 & 12-16 & 16-20 & 20-24 \\ 51.6 & 13.2 & 17.6 & 16.2 & 16 & 4.2 \end{bmatrix}$$

51.6 newborn

c. How many animals will be in the 12-16 month age group after 4 months?

16.2 12-16 month olds

d. How many animals will be born to the 16-20 month old mammals during this 4 month period?

16 16-20 month olds

e. Write the Leslie Matrix for this animal.

See L above!

f. What will be the total population after one cycle?

$$P_0 \cdot L \cdot C = 118.8 \text{ mammals}$$

g. What will be the total population after five cycles?

$$P_0 \cdot L^5 \cdot C = 159.82824 \text{ mammals}$$

h. What is the long-term growth rate for this mammal?

$$\frac{P_0 L^5 C - P_0 L^4 C}{P_0 L^5 C} = \frac{16.92864}{159.82824} = 10.59\% \text{ growth}$$

$$C = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$