Adv. Math Chapter 15

Review: Derivatives TWO

Name Key

Date____________ HR_____

1. Find the derivatives, $f'(x)$:

a. $f(x)=\frac{1}{2}x^2-x-2$

$$\boxed{f'(x)=x-1}$$

b. $f(x)=\frac{2x-7}{e^x}$

$$f'(x)=\frac{e^x(2)-(2x-7)(e^x)}{e^{2x}}$$

$$f'(x)=\frac{e^x(9-2x)}{e^{2x}}=\boxed{\frac{9-2x}{e^x}}$$

c. $f(x)=\frac{\ln x}{4x^2}$

$$f'(x)=\frac{4x^2\left(\frac{1}{x}\right)-\ln x(8x)}{16x^4}$$

$$=\frac{4x-8x\ln x}{16x^4}$$

$$=\frac{4x(1-2\ln x)}{4x(4x^3)}=\boxed{\frac{1-2\ln x}{4x^3}}$$

d. $f(x)=(2x-4)\sin x$

$$f'(x)=(2x-4)(\cos x)+\sin x(2)$$

$$=\boxed{2x\cos x-4\cos x+2\sin x}$$

e. $y=-x^3(3x^4-2)$

$$y'=-x^3(12x^3)+(3x^4-2)(-3x^2)$$

$$=-12x^6-9x^6+6x^2$$

$$=\boxed{-21x^6+6x^2}$$

f. $f(x)=\frac{5}{x^8}$

$$f'(x)=\frac{x^8(0)-5(8x^7)}{x^{16}}$$

$$=\frac{-40x^7}{x^{16}}=\boxed{\frac{-40}{x^9}}$$

g. $y=(-2x^4-3)(-2x^2+1)$

$$y'=(-2x^4-3)(-4x)+(-2x^2+1)(-8x^3)$$

$$y'=8x^5+12x+16x^5-8x^3$$

$$\boxed{y'=24x^5-8x^3+12x}$$

h. $f(x)=\sin 2x^3$

$u=2x^3$ $\quad f(u)=\sin u$

$u'=6x^2$ $\quad f'(u)=\cos u$

$$f'(x)=6x^2\cos u$$

$$=\boxed{6x^2\cos(2x^3)}$$

i. $f(x)=(5x^5+5)(-2x^5-3)$

$$f'(x)=(5x^5+5)(-10x^4)+(-2x^5-3)(25x^4)$$

$$=-50x^9-50x^4-50x^9-75x^4$$

$$=\boxed{-100x^9-125x^4}$$

J. $y=(-5x^3-3)^3$

$u=-5x^3-3$ $\quad f(u)=u^3$

$u'=-15x^2$ $\quad f'(u)=3u^2$

$$y'=-15x^2(3u)^2$$

$$y'=-15x^2(3(-5x^3-3))^2$$

$$y'=-15x^2(-15x^3-9)^2$$

$$y'=-15x^2(225x^9+270x^3+81)$$

$$\boxed{y'=-3375x^{11}-4050x^5-1215x^2}$$

Given the function $f(x) = 6x^7 - 9x^4 + 3x^2 + 2$, find the following.

$f'(x) =$ $42x^6 - 36x^3 + 6x$

$f''(x) =$ $252x^5 - 108x^2 + 6$

2. For each problem, find the equation of the tangent line at the given value.

a. $y = x^3 - 2x^2 + 2$ at x = 2

$y = (2)^3 - 2(2)^2 + 2$

$y = 2$ $\quad (2, 2)$

$y' = 3x^2 - 4x$

$y' = 3(2)^2 - 4(2)$

$= 12 - 8$

$= 4$ $\quad m = 4$

$\boxed{y - 2 = 4(x-2)}$

b. $y = -\dfrac{3}{x^2 - 25}$ at x = -4

$y = \dfrac{-3}{(-4)^2 - 25} = \dfrac{1}{3}$ $\quad \left(-4, \frac{1}{3}\right)$

$y' = \dfrac{(x^2-25)(0) - -3(2x)}{(x^2-25)^2}$

$y' = \dfrac{6x}{(x^2-25)^2}$

$y' = \dfrac{6(-4)}{((-4)^2-25)^2} = \dfrac{-24}{81} = \dfrac{-8}{27}$

$\boxed{y - \frac{1}{3} = \frac{-8}{27}(x+4)}$

c. $y = (5x+5)^{\frac{1}{2}}$ at x = 4

$u = 5x + 5 \quad f(u) = u^{1/2}$

$u' = 5 \quad f'(u) = \dfrac{\frac{1}{2}}{\sqrt{u}} = \dfrac{1}{2\sqrt{u}} \cdot \dfrac{\sqrt{u}}{\sqrt{u}} = \dfrac{\sqrt{u}}{2u}$

$y' = \dfrac{5\sqrt{5x+5}}{10x+10} = \dfrac{\sqrt{5x+5}}{2x+2}$

$(4, 5)$

$m = \frac{1}{2}$

$\boxed{y - 5 = \frac{1}{2}(x-4)}$