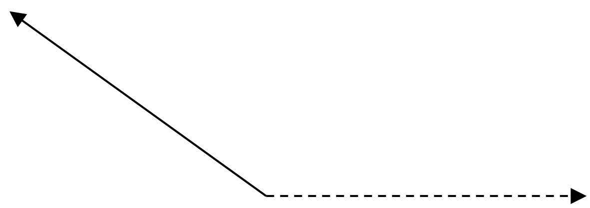
Section 8.1: Geometric Vectors

Objectives:
- I can identify the magnitude and direction of a vector.
- I can add/subtract vectors using the parallelogram method.
- I can add/subtract vectors using the "tip to tail" (triangle) method.
- I can find the component form of a vector.

Definition:
A **vector** is a quantity that is defined by its ______________ and ________________.

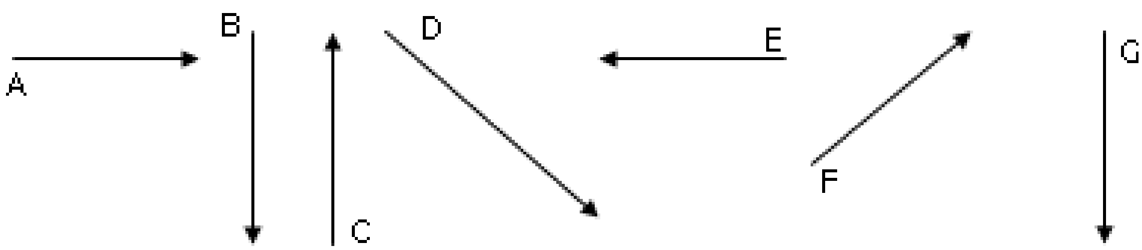
Example 1:
Use a ruler and protractor to determine the **magnitude** and **direction** of the vector below:



Definition:
A **zero vector** has a magnitude of 0 and can go in any direction (a point at the origin).

Example 2:
Draw a "zero vector".

Example 3:
Consider the vectors listed below. Use the vectors to solve the problems:



a. Name 2 vectors that are equal.

b. Name all of the parallel vectors.

c. Find the sum of **a** + **f** using the parallelogram and triangle methods.

Example 4:
Use the triangle method to find 2**c** – ½ **d**. (Use the pictures from example 3)

Example 5:
At a competition, a sailplane traveled forward at a rate of 8 m/s, and it descended at a rate of 4 m/s. Determine the magnitude of the resultant velocity of the sailplane.

Definition: Two or more vectors whose sum is a given vector are called **components** of the given vector.

Example 6:
A ship leaving port sails for 75 miles in a direction 35^o north of due east. Find the magnitude of the vertical and horizontal components.

Example 7:
A piling for a high-rise building is pushed by two bulldozers at exactly the same time. One bulldozer exerts a force of 1550 pounds in a westerly direction. The other bulldozer pushes the piling with a force of 3050 pounds in a northerly direction.

a. What is the magnitude of the resultant force upon the piling, to the nearest ten pounds?

b. What is the direction of the resultant force upon the piling, to the nearest tenth?

Section 8.2: Algebraic Vectors

Objectives:

- I can find ordered pairs that represent vectors (component form).
- I can Add, subtract, multiply, and find the magnitude of vectors algebraically.

Vectors as ordered pairs: Vectors can be represented using ordered pairs. For example, $\langle 3, 5\rangle$ can represent a vector in standard position. That means that the initial point is at (,) and the terminal point is at (,).

Definition: Two or more vectors whose sum is a given vector are called **components** of the given vector.
Given initial point ____________________ and terminal point ___________________, find the magnitude:

Example 1:

A) Draw $\langle 3,5\rangle$ in standard position and as a Triangle.

B) Shift the vector $\langle 3,5\rangle$ over 6 units to the left, and 1 unit down. Where are the new initial and terminal points?

Example 2:

Write the ordered pair that represents the vector from (-5, 6) to (4, -2). Then find the magnitude of the vector.

Definitions:

<u>Vector addition:</u> **a** + **b** =

<u>Vector subtraction:</u> **a** – **b** =

<u>Scalar multiplication:</u> k**a** =

<u>Example 3:</u>

Let **m** = $\langle 5, -7\rangle$, **n** = $\langle 0, 4\rangle$, and **p** = $\langle -1, 3\rangle$. Find each of the following:

a. **m** + **p**

b. **m** – **n**

c. 7**p**

d. 2**m** + 3**n** – **p**

<u>Example 4:</u>

Paramedics are moving a person on to a stretcher. Ms. Gonzalez is pushing the stretcher with a force of 135 newtons at 58° with the horizontal (from standard position). Mr. Howard is pulling the stretcher with a force of 214 newtons at 43° with the horizontal. What is the magnitude of the force exerted on the stretcher?

Definition:

A vector that has a magnitude of one unit is called a **unit vector**. A unit vector in the direction of the positive *x*-axis is represented by **i**, and the unit vector in the direction of the positive *y*-axis is represented by **j**.

So, **i =** $\langle \quad , \quad \rangle$and **j** = $\langle \quad , \quad \rangle$. The zero vector, therefore, would be represented as **0** = 0**i** + 0**j**.

<u>Example 5:</u>

Write **AB** as the sum of unit vectors for **A** = $(\mathbf{4}, \mathbf{-1})$ and **B** = $(\mathbf{6}, \mathbf{2})$.

You try it!!

1. Write the ordered pair that represents the vector from (7, -3) to (-2, -1). Then find the magnitude of the vector.

2. Let **u** = $\langle 1, -4 \rangle$ and **v** = $\langle 0, 8 \rangle$. Find each of the following:
   a. **u** + **v**

   b. **u** – **v**

   c. ½ **v**

   d. 2**u** + 3**v**

3. Radiology technicians are moving a patient on an MRI machine cot. Ms. Jones is pushing the cot with a force of 120 newtons at 55^{o} with the horizontal, while Mr. Michaels is pulling the cot with a force of 200 newtons at 40^{o} with the horizontal. What is the magnitude of the force exerted on the cot?

Section 8.5: Applications with Vectors

Objectives:

- Solve problems using vectors and right triangle trigonometry.

1. A bull-rider has finished his competition ride and the two rodeo clowns are restraining the bull to return it to the paddocks. Suppose one clown is exerting a force of 270 newtons due north and the other is pulling with a force of 360 newtons due east. What is the resultant force on the bull?
   a. Draw a labeled diagram that represents the forces.

   b. Determine the resultant force.

   c. Find the angle the resultant force makes with the east-west axis.

2. Tom works for a package delivery service. Suppose he is pushing a cart full of packages weighing 100 pounds up a ramp 8 feet long at an incline of 25°. Find the work done by gravity as the cart moves the length of the ramp. Assume that friction is not a factor.

Definition Review: What is the Law of Sines formula?

What is the Law of Cosines formula?

3. A lighting system for a theater is supported equally by two cables suspended from the ceiling of the theater. The cables form a 140^o angle with each other. If the lighting system weighs 950 pounds, what is the force exerted by each of the cables on the lighting system?

4. A Boeing 727 airplane, flying due east at 500 mph in still air, encounters a 70 mph tail wind acting in the direction 60^o north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What are they?

Section 8.4: Perpendicular (Orthogonal) Vectors

Objectives:

- I can find the inner (dot) product of two vectors.
- I can find the cross products of two vectors.
- I can determine whether two vectors are perpendicular (orthogonal).

The **dot product** of u = (u_1, u_2) and v = (v_1, v_2) is: $\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2$.

Note the answer is a number, not a vector.

Example 1:

Calculate the dot product of the vectors below:

a. **u** = (2, 3) and **v** = (7, -1).

b. **u** = (-5, 7) and **v** = (3, 5).

Example 2:

Let **u** = (3, -5), **v** = (-2, 4) and **w** = (6, 7). Find the following answers. (Be careful when deciding if the answer will be a *number* or a *vector*.)

a. $(\vec{u} \cdot \vec{v})\vec{w}$ =

b. $(\vec{u} + \vec{v}) \cdot \vec{w}$ =

The **Angle in Between** two vectors is: $\cos x = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$

Example 3:

Find the angle between the pairs of vectors in *Example 1*.

Orthogonal (perpendicular) Vectors:

Activity:

Work together to find a simple equation to determine if two vectors are perpendicular.

1. If two vectors are orthogonal (perpendicular), what is the measure of the angle, x, between the two vectors?

x = ________

cos x = _______

1. What is the cos x?

$$\cos x = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$$

2. Use the "cos x" you just found in #2, and the formula for the angle between to determine a NEW formula for perpendicular vectors.

Example 4:

Verify that the following vectors are perpendicular: $\langle 4,\ 2\rangle\,\langle 5,\ -10\rangle$

Parallel Vectors:

Example 5:
Verify that the following vectors are parallel: $\langle -3,\ 6\rangle\ \langle 5,\ -10\rangle$

Example 6:
Determine whether the following vectors are parallel, orthogonal, or neither.

#	Vectors	Work	Parallel, Orthogonal, or neither?
A	**u** = (8, 10) and **w** = (5, 4)		
B	**u = (**8, 10) and **w** = (1, 1.25)		
C	**u** = (8, 10) and **w** = (-4, -5)		
D	**u** = (8, 10) and **w** = (-20, 16)		
E	**u** = (8, 10) and **w** = (30, -24)		
F	**u** = (-2, 4, 8) and **w** = (16, 4, 2)		

Example 7:
Recall that perpendicular lines have opposite reciprocal slopes. Suppose that vector **u** = (3, 4).

a. Find two vectors that are orthogonal (perpendicular) to **u**.

b. “Scale-change” your answers from part (a) so that the vectors have a length of 10.

Section 8.6: Vectors and Parametric Equations

Objectives:

- Write parametric equations of lines.
- Graph parametric equations.

Recall:

Slope Intercept Form: $y = mx + b$

Point-Slope Form: $y - y_1 = m(x - x_1)$

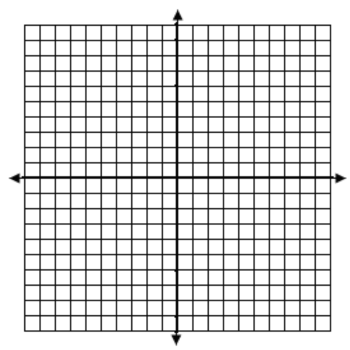
Definition (Parametric equation of a line):

A line through $P_1\ (x_1, y_1)$ that is parallel to the vector $\vec{v} = (v_1, v_2)$

$$x = x_1 + tv_1$$

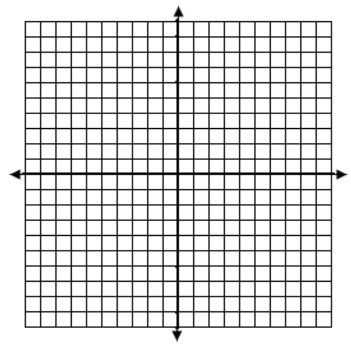
$$y = y_1 + tv_2$$

1. Consider the vector **v**.
   a. Sketch the vector that is represented by the vector from (-2, 5) to (7, -3).
   b. Give the component representation of the standard position vector.

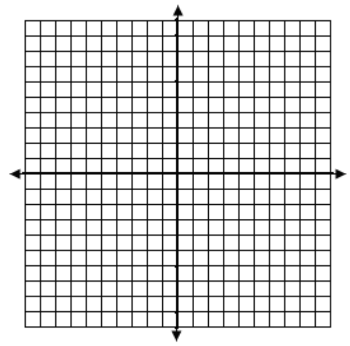


   c. Sketch the vector in standard position.
   d. Find the magnitude and direction of **v**.

2. Consider **v** = (3, 7).
   a. Sketch **v**.
   b. Sketch **u** such that **u** starts at (-5, 8) and is parallel to vector **v**.



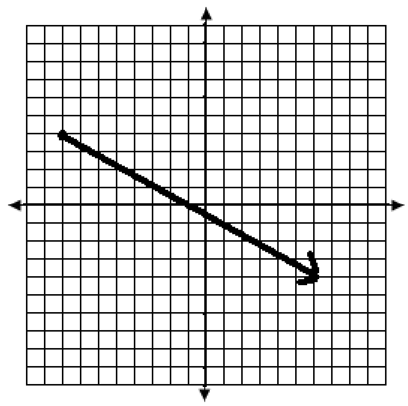
3. Vector **v** has a length of 6 and is in the direction of 45° west of north.
   a. Give the component representation of the vector.
   b. Sketch **v** with an initial point of (5, -3).
   c. Identify the endpoint of the vector drawn in part b.



4. Consider the vector drawn below:
   a. Give the initial point.

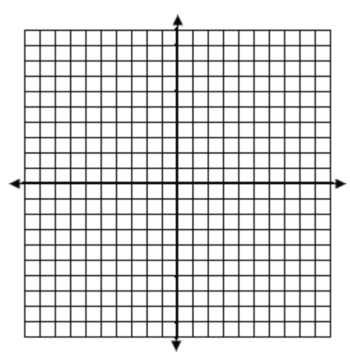
   b. Give the terminal point.

   c. Give the component representation of the vector in standard form.



5. Consider the line given by the parametric equations below. Graph the line.

$$x = -8 + 12t$$
$$y = -3 + 7t$$



   a. Sketch a new line that contains the point (-4, 1) and is parallel to the line above.

   b. Write the equation for this line in **parametric form.**

   c. Write the equation of the line in **point-slope form**.

6. A vector **w** = (w_1, w_2) in standard position is parallel to ***u*** = (3, 1) and twice as long. Find two sets of values for (w_1, w_2).

7. Find a vector **w** = (w_1, w_2) in standard position that is parallel to **v** = (3, -4) and has a length of 30.

8. Find a vector **w** = (w_1, w_2) in standard position that is parallel to **u** = (-5, -12) and has a length of 32.5.

9. Write a vector that is perpendicular (orthogonal) to the vector (6, -5).

10. Write a vector that is perpendicular (orthogonal) to the line with parametric equations:

$$\begin{aligned} x &= -5 + 8t \\ y &= 9 - 3t \end{aligned}.$$

11. Graph $\begin{aligned} x &= 8 - 10t \\ y &= 4 + 3t \end{aligned}$.