

## CHAPTER 7

### Section 7.1 Notes

Objectives:

- Identify and use the reciprocal identities, quotient identities, Pythagorean identities, symmetry identities, and opposite-angle identities.

#### **Example 1:**

Prove that  $\sin x \cos x = \tan x$  is NOT a trigonometric identity by producing a counterexample.

*Check it out!!!!!! Look on your "Formula" Sheet and find the following identities:*

Reciprocal Identities

Quotient Identities

Pythagorean Identities

#### **Example 2:**

Use the information to find the trigonometric value:

a. If  $\sec x = \frac{3}{2}$ , find  $\cos x$ .

b. If  $\csc x = \frac{4}{3}$ , find  $\tan x$ .

**Example 3:**

Express each value as a trig function of an angle in Quadrant I (Reference angle).

a.  $\sin 600$

b.  $\sin \frac{19\pi}{4}$

c.  $\cos(-410)$

d.  $\tan \frac{37\pi}{6}$

*Check it out!!!!!! Look on your "Formula" Sheet and find the following identities:*

Opposite Angle Identities

**Example 4:**

Simplify:  $\sin x + \sin x \cot^2 x$

## Section 7.2 (Day 1)

Objectives:

- Use the basic trig identities to verify other identities.

Example 1:

Prove that  $\sec^2 x - \tan x \cot x = \tan^2 x$  is an identity (both sides are equal).

What is the domain of the identity?

Example 2:

Prove that  $\frac{\sin A}{\csc A} + \frac{\cos A}{\sec A} = \csc^2 A - \cot^2 A$  is an identity.

State the domain of the identity.

**Section 7.2 (Day 2):**

Objective:

- Find numerical values of trig functions.

Example 1:

Use trig. identities to prove:  $\frac{\cos(A + 360)}{\cos(360 - A)} = \cos A \sec A$ . State the domain of the identity.

Example 2:

Find a numerical value of one trigonometric function of  $x$  if  $\frac{\cot x}{\cos x} = 2$ .

**Mixed Review:**

1. State if the following is an identity or not. If not, give a counterexample. If so, prove it.

a.  $(\cos x + \sin x)^2 = \cos^2 x + \sin^2 x$

b.  $\cos x \tan x = \sin x$

2. Prove:  $1 + \tan^2 x = \sec^2 x$  and state the domain of the function.

3. If  $\csc x = \frac{-5}{4}$  in the third quadrant, find  $\tan x$ .

## Section 7.3/7.4 Notes

Objectives:

- Use the sum and difference identities for the sine, cosine, and tangent functions.

*Check it out!!!!!! Look on your "Formula" Sheet and find the following identities:*

Difference of cosines

Sum of cosines

Difference of sines

Sum of sines

Example 1:

Use the sum or difference identity to find the EXACT VALUE of  $\cos 735^\circ$ .

Example 3:

Use the sum or difference identity to find the EXACT VALUE of  $\sin \frac{7\pi}{12}$ .

Example 4:

Find the value of  $\sin(x - y)$  if  $0 < x < \frac{\pi}{2}$ ,  $0 < y < \frac{\pi}{2}$ ,  $\sin x = \frac{9}{41}$ , and  $\sin y = \frac{7}{25}$ .

Try these with your partner:

1. Use the sum or difference identity to find an EXACT value of  $\cos 75^\circ$ .
2. Find the value of  $\sin(x + y)$  if  $0 < x < \frac{\pi}{2}$ ,  $0 < y < \frac{\pi}{2}$ ,  $\sin x = \frac{4}{5}$ , and  $\cos y = \frac{5}{13}$ .
3. Verify that  $\sec(\pi + A) = -\sec A$  is an identity.

## Section 7.4 Notes

Objectives:

- Use the double angle identity for the sine and cosine functions.

*Check it out!!!!!! Look on your "Formula" Sheet and find the following identity:*

### Double Angle Identities

Example 1:

If  $\sin x = \frac{2}{3}$ , and  $x$  has its terminal side in the **first quadrant**, find the EXACT value of each function.

a.  $\sin 2x$

b.  $\cos 2x$



## Section 7.5 Notes

Objectives:

- Solve trigonometric equations and inequalities.

Recall the range of the functions:  $\cos^{-1}x$ ,  $\sin^{-1}x$ , and  $\tan^{-1}x$ . In which quadrants are each of those functions defined?

The answers in the quadrants above are called the **principal values** of the trig functions.

Example 1:

Solve  $\sin x \cos x - \frac{1}{2} \cos x = 0$  for principal values of  $x$ . Express your solutions in degrees and radians.

Example 2:

a. Solve  $\cos^2 x - \cos x + 1 = \sin^2 x$  for  $0 \leq x < 2\pi$ .

b. What would your solution be over the set of all real numbers?

Example 3:

Solve  $2\sec^2 x - \tan^4 x = -1$  for all real values of  $x$ .

Example 4:

Solve  $2\sin x + 1 > 0$  for  $0 \leq x < 2\pi$ .

## Chapter 7 Formula Sheet

### Reciprocal Identities

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

### Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

### Opposite Angle Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

### Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

### Sum and Difference Identities

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

### Double-Angle Identities

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 1 - 2 \sin^2 x$$

$$\cos 2x = 2 \cos^2 x - 1$$

### Symmetry Identities

$$\sin(x + 360) = \sin x$$

$$\cos(x + 360) = \cos x$$

$$\sin(x + 180(2k - 1)) = -\sin x$$

$$\cos(x + 180(2k - 1)) = -\cos x$$

$$\sin(360 - x) = -\sin x$$

$$\cos(360 - x) = \cos x$$

$$\sin(180(2k - 1) - x) = \sin x$$

$$\cos(180(2k - 1) - x) = -\cos x$$