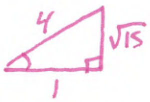
Name Key Date ________ Hr. ________

7-1 Worksheet: Basic Trig Identities

Use the given information to determine the exact trigonometric value if $0^0 < \theta < 90^0$

1). $\cos\theta = \frac{1}{4}$, find $\tan\theta$



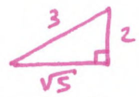
$\tan\theta = \frac{\sqrt{15}}{1} = \boxed{\sqrt{15}}$

$a^2 + 1^2 = 4^2$

$a^2 = 15$

$a = \sqrt{15}$

2). If $\sin\theta = \frac{2}{3}$, find $\cos\theta$



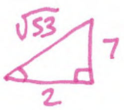
$\cos\theta = \boxed{\frac{\sqrt{5}}{3}}$

$a^2 + 2^2 = 3^2$

$a^2 = 5$

$a = \sqrt{5}$

3). If $\tan\theta = \frac{7}{2}$, find $\sin\theta$



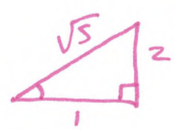
$\sin\theta = \frac{7}{\sqrt{53}} \cdot \frac{\sqrt{53}}{\sqrt{53}} = \boxed{\frac{7\sqrt{53}}{53}}$

$2^2 + 7^2 = c^2$

$c^2 = 53$

$c = \sqrt{53}$

4). If $\tan\theta = 2$, find $\cot\theta$



$\cot\theta = \boxed{\frac{1}{2}}$

$1^2 + 2^2 = c^2$

$c^2 = 5$

$c = \sqrt{5}$

Express each value as a trigonometric function of an angle in Quadrant I.

5). $\cos 892^{\circ}$

$-\cos 8$

6). $\csc 495^{\circ}$

$\csc 45$

7). $\sin \frac{23\pi}{3}$

$-\sin \frac{\pi}{3}$

Simplify each expression.

8). $\cos x + \sin x \tan x$

$\cos x + \sin x \left(\frac{\sin x}{\cos x}\right)$

$\cos x + \frac{\sin^2 x}{\cos x}$

$\frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x}$

$\frac{1}{\cos x}$

$\boxed{\sec x}$

9). $\frac{\cot A}{\tan A}$

$\frac{\cos A}{\sin A} \cdot \frac{\cos A}{\sin A}$

$\frac{\cos^2 A}{\sin^2 A}$

$\boxed{\cot^2 A}$

Name Key Date ______________ Hr. ________

7-2 Worksheet: Proving Trig Identities

Verify that each equation is an identity and state the domain of the identity.

1). $\dfrac{\csc x}{\cot x + \tan x} = \cos x$

$$\frac{\frac{1}{\sin x}}{\frac{\cos x \cdot \cos x}{\cos x \cdot \sin x} + \frac{\sin x \cdot \sin x}{\cos x \cdot \sin x}} = \cos x$$

$$\frac{\frac{1}{\sin x}}{\frac{\cos^2 x + \sin^2 x}{\cos x \sin x}} = \cos x$$

$$\frac{\frac{1}{\sin x}}{\frac{1}{\cos x \sin x}} = \cos x$$

$$\frac{1}{\sin x} \cdot \frac{\cos x \sin x}{1} = \cos x$$

$$\boxed{\cos x = \cos x}$$

$D: \{x \neq \pm\frac{\pi}{2}n\}$

2). $\dfrac{1}{\sin x - 1} - \dfrac{1}{\sin x + 1} = -2\sec^2 x$

$$\frac{(\sin x+1)}{(\sin x+1)}\frac{1}{\sin x - 1} - \frac{1}{\sin x + 1}\frac{(\sin x - 1)}{(\sin x - 1)} = -2\sec^2 x$$

$$\frac{\sin x + 1}{\sin^2 x - 1} - \frac{\sin x - 1}{\sin^2 x - 1} = -2\sec^2 x$$

$$\frac{2}{\sin^2 x - 1} = -2\sec^2 x$$

$$\frac{2}{-\cos^2 x} = -2\sec^2 x$$

$$-2\left(\frac{1}{\cos^2 x}\right) = -2\sec^2 x$$

$$\boxed{-2\sec^2 x = -2\sec^2 x}$$

$D: \{x \neq \frac{\pi}{2} \pm \pi n\}$

3). $\sin^3 x - \cos^3 x = (1 + \sin x \cos x)(\sin x - \cos x)$

$$\sin^3 x - \cos^3 x = \sin x - \cos x + \sin^2 x \cos x - \sin x \cos^2 x$$

$$\sin^3 x - \cos^3 x = \sin x - \cos x + (1 - \cos^2 x)(\cos x) - \sin x(1 - \sin^2 x)$$

$$\sin^3 x - \cos^3 x = \sin x - \cos x + \cos x - \cos^3 x - \sin x + \sin^3 x$$

$$\sin^3 x - \cos^3 x = -\cos^3 x + \sin^3 x$$

$$\boxed{\sin^3 x - \cos^3 x = \sin^3 x - \cos^3 x}$$

$D: \{\mathbb{R}\}$

4). $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$

$$\frac{(1+\sin x)}{(1+\sin x)}\frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x}\frac{(\cos x)}{(\cos x)} = \sec x$$

$$\frac{\sin x + \sin^2 x}{\cos x + \sin x \cos x} + \frac{\cos^2 x}{\cos x + \sin x \cos x} = \sec x$$

$$\frac{\sin x + 1}{\cos x + \sin x \cos x} = \sec x$$

$$\frac{\sin x + 1}{\cos x\,(1+\sin x)} = \sec x$$

$$\frac{1}{\cos x} = \sec x$$

$$\boxed{\sec x = \sec x}$$

$D: \{x \neq \frac{\pi}{2} \pm \pi n\}$

Find a numerical value of one trigonometric function of x.

5). $\sin x \cot x = 1$

$$\sin x \cdot \frac{\cos x}{\sin x} = 1$$

$$\boxed{\cos x = 1}$$

6). $\sin x = 3\cos x$

$$\frac{\sin x}{\cos x} = \frac{3\cos x}{\cos x}$$

$$\boxed{\tan x = 3}$$

7). $\cos x = \cot x$

$$(\sin x)\cos x = \frac{\cos x}{\sin x}(\sin x)$$

$$\frac{\sin x \cos x}{\cos x} = \frac{\cos x}{\cos x}$$

$$\boxed{\sin x = 1}$$

Name Key Date ________ Hr. ________

7-3 Worksheet: Sum and Difference Identities

Use the sum and difference identities to find the exact value of each trig function.

1). $\cos\frac{5\pi}{12} = \cos\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right)$

$= \cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

2). $\sin(-165^0) = \sin(-30-135)$

$= \sin(-30)\cos(135) - \cos(-30)\sin(135)$

$= \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$= \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

3). $\tan(-\frac{7\pi}{12}) = \tan\left(\frac{-4\pi}{12}-\frac{3\pi}{12}\right)$

$\sin\left(\frac{-4\pi}{12}-\frac{3\pi}{12}\right)$

$= \sin\left(-\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(-\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{-\sqrt{6}-\sqrt{2}}{4}$

$\cos\left(-\frac{\pi}{3}-\frac{\pi}{4}\right)$

$\cos\left(-\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

$\frac{-\sqrt{6}-\sqrt{2}}{4} \cdot \frac{4}{\sqrt{2}-\sqrt{6}} = \frac{-\sqrt{6}-\sqrt{2}}{\sqrt{2}-\sqrt{6}} \frac{(\sqrt{2}+\sqrt{6})}{(\sqrt{2}+\sqrt{6})}$

$= \frac{-2-2\sqrt{12}-6}{2-6}$

$= \frac{-8-4\sqrt{3}}{-4}$

$= \boxed{2+\sqrt{3}}$

4). $\sec\frac{\pi}{12}$

$\cos\left(\frac{3\pi}{12}-\frac{2\pi}{12}\right)$

$\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$

$\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

$\frac{1}{\frac{\sqrt{6}+\sqrt{2}}{4}}$

$1 \cdot \frac{4}{\sqrt{6}+\sqrt{2}}$

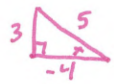
$\frac{4}{\sqrt{6}+\sqrt{2}} \frac{(\sqrt{6}-\sqrt{2})}{(\sqrt{6}-\sqrt{2})}$

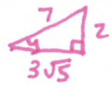
$\frac{4\sqrt{6}-4\sqrt{2}}{6-2} = \frac{4\sqrt{6}-4\sqrt{2}}{4} = \boxed{\sqrt{6}-\sqrt{2}}$

QI QII

Find each exact value if $0 < y < \frac{\pi}{2}$, $\frac{\pi}{2} < x < \pi$

5). $\cos(x+y)$ if $\sin x = \frac{3}{5}$, $\sin y = \frac{2}{7}$



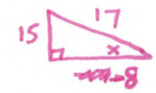


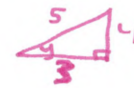
$\left(-\frac{4}{5}\right)\left(\frac{3\sqrt{5}}{7}\right) - \left(\frac{3}{5}\right)\left(\frac{2}{7}\right)$

$\frac{-12\sqrt{5}}{35} - \frac{6}{35}$

$\boxed{\frac{-12\sqrt{5}-6}{35}}$

6). $\sin(x-y)$ if $\cos x = -\frac{8}{17}$ and $\cos y = \frac{3}{5}$





$\left(\frac{15}{17}\right)\left(\frac{3}{5}\right) - \left(\frac{-8}{17}\right)\left(\frac{4}{5}\right)$

$\frac{45}{85} + \frac{32}{85}$

$\boxed{\frac{77}{85}}$

Verify that each equation is an identity.

7). $\cos(180^0 - \theta) = -\cos\theta$

$\cos 180 \cos\theta + \sin 180 \sin\theta = -\cos\theta$

$-1 \cos\theta + 0 \cdot \sin\theta = -\cos\theta$

$\boxed{-\cos\theta = -\cos\theta}$

8). $\sin(360^0 + \theta) = \sin\theta$

$\sin 360 \cos\theta + \cos 360 \sin\theta = \sin\theta$

$0 \cdot \cos\theta + 1 \sin\theta = \sin\theta$

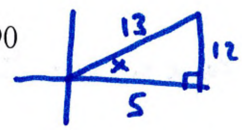
$\boxed{\sin\theta = \sin\theta}$

Name Key Date ________________ Hr. ________

7-4 Worksheet: Double-Angle Identities

Use the given information to find sin2x, cos2x

1). $\sin x = \frac{12}{13}$, for $0 < x < 90$



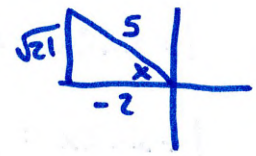
$\sin 2x = 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right) = \boxed{\frac{120}{169}}$

$\cos 2x = \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$

$= \frac{25}{169} - \frac{144}{169}$

$= \boxed{\frac{-119}{169}}$

2). $\sec x = -\frac{5}{2}$, for $\frac{\pi}{2} < x < \pi$



$\sin 2x = 2\left(\frac{\sqrt{21}}{5}\right)\left(-\frac{2}{5}\right)$

$= \boxed{\frac{-4\sqrt{21}}{25}}$

$\cos 2x = 2\left(-\frac{2}{5}\right)^2 - 1$

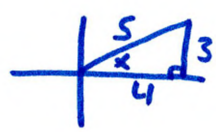
$= 2\left(\frac{4}{25}\right) - 1$

$= \frac{8}{25} - 1$

$= \boxed{\frac{-17}{25}}$

Use the given information to find sin2x, cos2x

3). $\sin x = \frac{3}{5}$, for $0 < x < \frac{\pi}{2}$



$\sin 2x = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$= \boxed{\frac{24}{25}}$

$\cos 2x = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \boxed{\frac{7}{25}}$

Verify that each equation is an identity.

4). $1+\sin 2x = (\sin x + \cos x)^2$

$1 + \sin 2x = \sin^2 x + 2\sin x \cos x + \cos^2 x$

$1 + \sin 2x = 1 + 2\sin x \cos x$

$1 + \sin 2x = 1 + \sin 2x$

5). $\cos x \sin x = \dfrac{\sin 2x}{2}$

$\cos x \sin x = \dfrac{2\sin x \cos x}{2}$

$\cos x \sin x = \sin x \cos x$

$\cos x \sin x = \cos x \sin x$

Name Key Date ________ Hr. ________

7-5 Worksheet: Solving Trigonometric Equations

Solve each equation for principal values of x. Express solutions in degrees.

1). $\cos x = 3\cos x - 2$

$-2\cos x = -2$

$\cos x = 1$

$\boxed{x = 0^\circ}$

2). $2\sin^2 x - 1 = 0$

$2\sin^2 x = 1$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$

$\sin x = \frac{\sqrt{1}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\boxed{x = 45^\circ}$

Solve each equation for $0 \leq x < 360$

3). $\sin^2 x - 2\sin x + 1 = 0$

$(\sin x - 1)(\sin x - 1) = 0$

$\sin x - 1 = 0$

$\sin x = 1$

$\boxed{x = \frac{\pi}{2} \text{ or } 90^\circ}$

4). $\cos 2x + 3\cos x - 1 = 0$

$2\cos^2 x - 1 + 3\cos x - 1 = 0$

$2\cos^2 x + 3\cos x - 2 = 0$

-4

-1 4

3

	$2\cos x$	-1
$\cos x$	$2\cos^2 x$	$-\cos x$
2	$4\cos x$	-2

$(\cos x + 2)(2\cos x - 1) = 0$

$\cos x + 2 = 0$

$\cancel{\cos x = -2}$

$2\cos x - 1 = 0$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$\boxed{x = 60^\circ, 300^\circ}$

Solve each equation for $0 \le x < 2\pi$

5). $4\sin^2 x - 4\sin x + 1 = 0$

	$2\sin x$	-1
$2\sin x$	$4\sin^2 x$	$-2\sin x$
-1	$-2\sin x$	1

$(2\sin x - 1)(2\sin x - 1) = 0$

$2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

6). $\cos 2x + \sin x = 1$

$1 - 2\sin^2 x + \sin x - 1 = 0$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$\sin x = 0$

$\boxed{x = 0, \pi}$

$2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

Solve each equation for all real values of x.

7). $3\cos 2x - 5\cos x = 1$

$3(2\cos^2 x - 1) - 5\cos x = 1$

$6\cos^2 x - 5\cos x - 4 = 0$

	$3\cos x$	-4
$2\cos x$	$6\cos^2 x$	$-8\cos x$
1	$3\cos x$	-4

$(2\cos x + 1)(3\cos x - 4) = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

$\boxed{\frac{2\pi}{3} \pm 2\pi n}$

$3\cos x = 4$

$\cos x = \frac{4}{3}$

$\boxed{\frac{4\pi}{3} \pm 2\pi n}$

8). $2\sin^2 x - 5\sin x + 2 = 0$

	$2\sin x$	-1
$\sin x$	$2\sin^2 x$	$-\sin x$
-2	$-4\sin x$	2

$(\sin x - 2)(2\sin x - 1) = 0$

~~$\sin x = 2$~~

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$\boxed{\frac{\pi}{6} \pm 2\pi n \quad \frac{2\pi}{3} \pm 2\pi n}$

9). $3\sec^2 x - 4 = 0$

$3\left(\frac{1}{\cos^2 x}\right) = 4$

$3 = 4\cos^2 x$

$\frac{3}{4} = \cos^2 x$

$\cos x = \frac{\sqrt{3}}{2}$

$\boxed{\frac{\pi}{6} \pm 2\pi n \quad \frac{11\pi}{6} \pm 2\pi n}$

10). $\tan x(\tan x - 1) = 0$

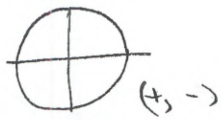
$\tan x = 0$ $\quad$ $\tan x = 1$

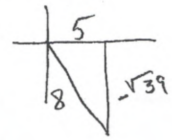
$\boxed{\pm \pi n \quad \frac{\pi}{4} \pm \pi n}$

Advanced Math **Name** ______________

Chapter 7 review Part 1

1. Given $\frac{3\pi}{2} \le x \le 2\pi$ and $\cos x = \frac{5}{8}$ find the following:





a. $\sin x = \dfrac{-\sqrt{39}}{8}$

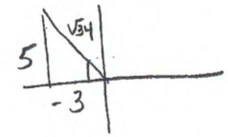
b. $\csc x = \dfrac{-8\sqrt{39}}{39}$

c. $\tan x = \dfrac{-\sqrt{39}}{5}$

d. $\sec x = 8/5$

e. $\cot x = \dfrac{-5\sqrt{39}}{39}$

2. Given $90° \le x \le 180°$ and $\tan x = \frac{-5}{3}$ find the following:



$\cos \dfrac{-3}{\sqrt{34}} = \dfrac{-3\sqrt{34}}{34}$

a. $\cos 2x = 2\cos^2 x - 1$

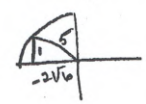
$2\left(\frac{-3}{\sqrt{34}}\right)^2 - 1$

$2\left(\frac{9}{34}\right) - 1 = \frac{18}{34} - \frac{34}{34} = \frac{-16}{34} = \frac{-8}{17}$

b. $\sin 2x = 2\cos x \sin x$

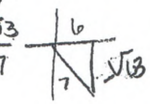
$2\left(\frac{-3\sqrt{34}}{34}\right)\frac{5\sqrt{34}}{34} = \frac{-30 \cdot 34}{34 \cdot 34} = \frac{-1020}{1156}$

3. Given: $\frac{\pi}{2} \le x \le \pi$ and $\sin x = \frac{1}{5}$. Given: $\frac{3\pi}{2} \le y \le 2\pi$ and $\cos y = \frac{6}{7}$.



$\cos x = \dfrac{-2\sqrt{6}}{5}$

$\sin y = \dfrac{-\sqrt{13}}{7}$



a. Find $\cos(x - y) = \cos x \cos y + \sin x \sin y$

$\frac{-2\sqrt{6}}{5} \cdot \frac{6}{7} + \frac{1}{5} \cdot \frac{-\sqrt{13}}{7}$

$= \dfrac{-12\sqrt{6} - \sqrt{13}}{35}$

b. Find $\sin(x + y) = \sin x \cos y + \sin y \cos x$

$\frac{1}{5} \cdot \frac{6}{7} + \frac{-\sqrt{13}}{7} \cdot \frac{-2\sqrt{6}}{5}$

$\frac{6}{35} + \frac{2\sqrt{78}}{35} = \dfrac{6 + 2\sqrt{78}}{35}$

4. Find $\sin(\overset{255^\circ}{\cancel{265^\circ}}) = \sin(225 + 30^\circ) = \sin 225 \cos 30 + \sin 30 \cos(225)$

$$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2}\left(\frac{-\sqrt{2}}{2}\right)$$

$$\frac{-\sqrt{6} - \sqrt{2}}{4}$$

5. Find $\cos\left(\frac{7\pi}{12}\right) = \cos(105^\circ) = \cos(45 + 60)$

$$= \cos 45 \cos 60 - \sin 45 \sin 60$$

$$\frac{\sqrt{2}}{2} \quad \frac{1}{2} \quad - \quad \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

6. Find $\cos\left(\frac{3\pi}{8}\right) = \cos\left(\frac{3\pi/4}{2}\right) = \pm\sqrt{\frac{1 + \cos 3\pi/4}{2}}$

$$= \pm\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \pm\sqrt{\frac{2 - \sqrt{2}}{2} \cdot \frac{1}{2}} = \pm\sqrt{\frac{2 - \sqrt{2}}{4}} = \pm\frac{\sqrt{2 - \sqrt{2}}}{2}$$

7. $\sin(105^\circ) = \sin(45 + 60)$

$$= \sin 45 \cos 60 + \cos 45 \sin 60$$

$$\frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

8. $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi/6}{2}\right)$

$$= \pm\sqrt{\frac{1 - \cos \pi/6}{2}} = \pm\sqrt{\frac{1 - \sqrt{3}/2}{2}} = \pm\sqrt{\frac{\frac{2}{2} - \frac{\sqrt{3}}{2}}{2}}$$

$$= \pm\sqrt{\frac{2 - \sqrt{3}}{2} \cdot \frac{1}{2}} = \pm\sqrt{\frac{2 - \sqrt{3}}{4}} = \pm\frac{\sqrt{2 - \sqrt{3}}}{2}$$

Advanced Math

Chapter 7 review Part 2

Solve each equation for principal values of x.

1. $2\sin x + 1 = 0$

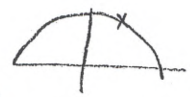
$\sin x = -\frac{1}{2}$

$x = -30^\circ$

$x = -\frac{\pi}{6}$

$x = 60^\circ$

$x = \frac{\pi}{3}$



2. $2\cos^2 x + 3\cos x = 2$

$2\cos^2 x + 3\cos x - 2 = 0$

$2y^2 + 3y - 2 = 0$

$(2y - 1)(y + 2) = 0$

$(2\cos x - 1)(\cos x + 2) = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -2$ NOPE

Solve each equation for the interval $0 \leq x \leq 2\pi$.

3. $4\sin^2 x + 1 = -4\sin x$

$4\sin^2 x + 4\sin x + 1 = 0$

$(2\sin x + 1)(2\sin x + 1) = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

4. $\cos x \tan x - 2\cos^2 x = -1$

$\cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} - 2\cos^2 x = -1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin x = 2\cos^2 x - 1$

$\sin x = 1 - 2\sin^2 x$ IDENTIES

$\sin x = -1$

$x = \frac{3\pi}{2}$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

Solve each equation where the Domain of x is all real #'s.

5. $3\cos^2 x = 6\cos x - 3$

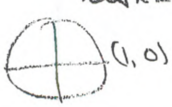
$3\cos^2 x - 6\cos x + 3 = 0$

$3(y^2 - 2y + 1) = 0$

$3(y - 1)(y - 1) = 0$

$3(\cos x - 1)(\cos x - 1) = 0$

$\cos x = 1$ $\quad$ $x = \pm 2\pi n$



(1, 0)

6. $4\sin^2 x - 2 = 0$

$2(2\sin^2 x - 1) = 0$

$2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4} \pm \frac{\pi}{2} n$

Verify that the following trigonometric expressions are identities. State the domain restrictions if any.

1. $\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = \sec^2 x - \tan^2 x$

$\cos^2 x + \sin^2 x = \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}$

$= \frac{1 - \sin^2 x}{\cos^2 x}$

$1 = \frac{\cos^2 x}{\cos^2 x}$

$1 = 1$

Domain: $x \neq \frac{\pi}{2} \pm \pi n$

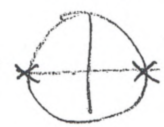
2. $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = 2\csc^2 x$

$\frac{1+\cos x}{1-\cos^2 x} + \frac{1-\cos x}{1-\cos^2 x} = \frac{2}{1-\cos^2 x} = \frac{2}{\sin^2 x} = \boxed{2\csc^2 x = 2\csc^2 x}$

Domain: $\cos x \neq \pm 1$

$\sin x \neq 0$

D: $\{x \neq \pm\pi n$



3. $\frac{\sin\theta}{\csc\theta}+\frac{\cos\theta}{\sec\theta}=\sin\theta\csc\theta$

$$\frac{\sin\theta}{\frac{1}{\sin\theta}} + \frac{\cos\theta}{\frac{1}{\cos\theta}} = \sin^2\theta + \cos^2\theta$$

$$= 1$$

So $1 = \sin\theta \cdot \csc\theta$

$1 = \sin\theta \cdot \frac{1}{\sin\theta}$

$1 = 1$

Domain: $D: \{x \neq \pm \pi/2\, n$

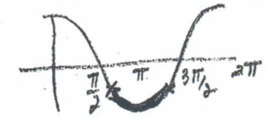
4. $\frac{\tan^2\theta}{1+\tan^2\theta}=\sin^2\theta$

$$\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta} = \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} = \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{1} = \sin^2\theta$$

$$\boxed{\sin^2\theta = \sin^2\theta}$$

Domain: $D: \{x \neq \pm \pi n$

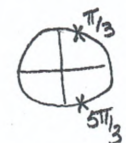
Solve the following inequalities. Domain $0 \le x \le 2\pi$. Hint... Draw the circle and the cosine or sine wave ☺.

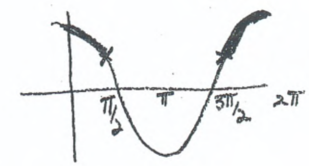


1. $\cos x \le \frac{-\sqrt{3}}{2}$

$5\pi/6 \le x \le 7\pi/6$

2. $\cos x - \frac{1}{2} > 0$ $\quad \cos x > 1/2$





$0 \le x \le \pi/3$

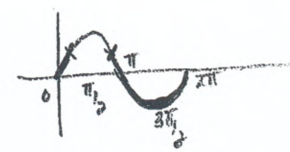
$5\pi/3 \le x \le 2\pi$

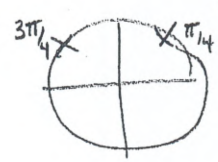
3. $\sqrt{2}\,\sin x - 1 < 0$

$\sqrt{2}\,\sin x < 1$

$\sin x < \frac{1}{\sqrt{2}}$

$\sin x < \frac{\sqrt{2}}{2}$





$0 \le x < \pi/4$

$3\pi/4 < x \le 2\pi$