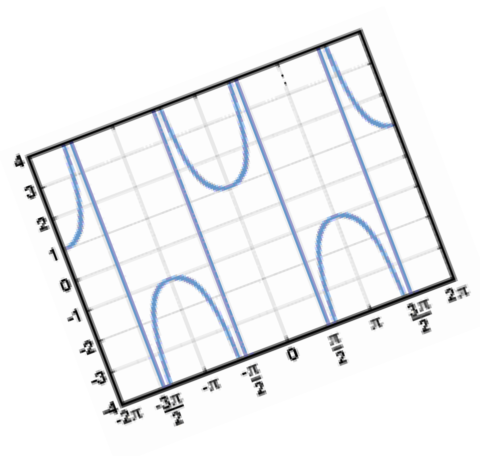
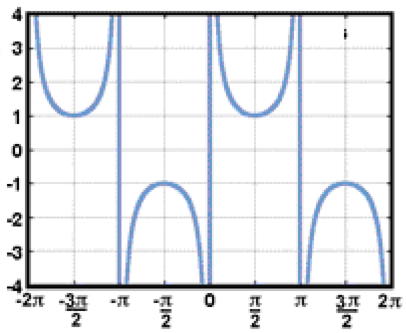
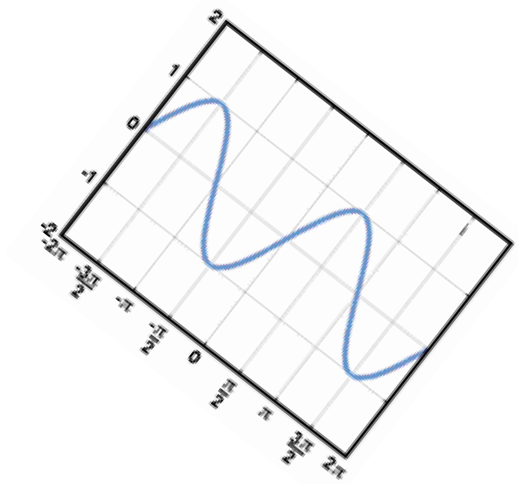
Trig Graphing Packet #1

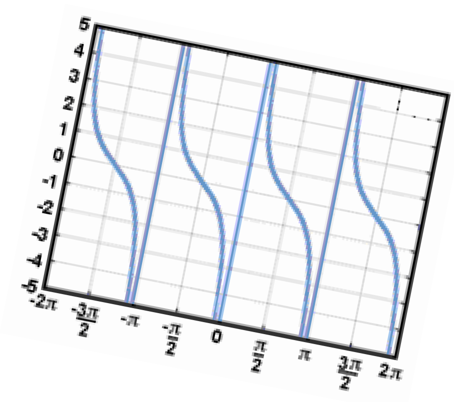
Chapter 6 – Advanced Math

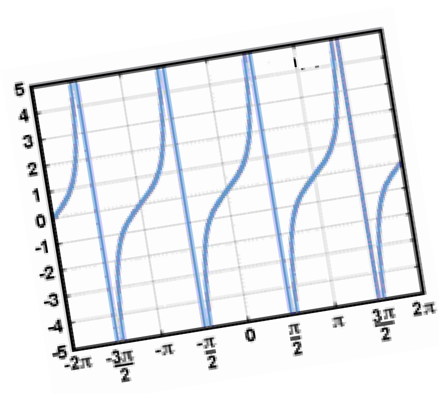
…..The basics…..











Please note, what you are about to do is a QUALITY over QUANTITY assignment, you do not win an award for how quickly you finish it. You will not benefit from dividing up the work. Please, each student should do each step!!!!!

Learning Targets:	Tasks:
• I can graph the sin, cos, and tangent functions. • I can graph the csc, sec, and cot functions.	• Create a table of values for the trig function • Graph the trig function. • Identify certain properties and characteristics of that function.

Steps:

1. Starting with sine (and looking at the sine page). Fill in the T-chart with the sin values for $0 < \theta < 2\pi$.

2. Based on your answers from the chart, begin graphing the sin function WITHOUT A CALCULATOR. *Be careful, and use a PENCIL for this!*

3. Once you have an accurate graph for the sine function from $0 < \theta < 2\pi$, copy the graph backwards so that you now have the entire graph, $-2\pi < \theta < 2\pi$.

4. Answer the questions about sine that follow. It may help to draw "mini-circles" to show/think about each answer.

5. Check your sine graph on your calculator. *Make sure you are in radians and you have changed your window to something compatible with radians!*

6. With your group, discuss the connections between the *questions in #4* and the *graphs in #2 and #3*. Doing this will help you fill out the "properties" box.

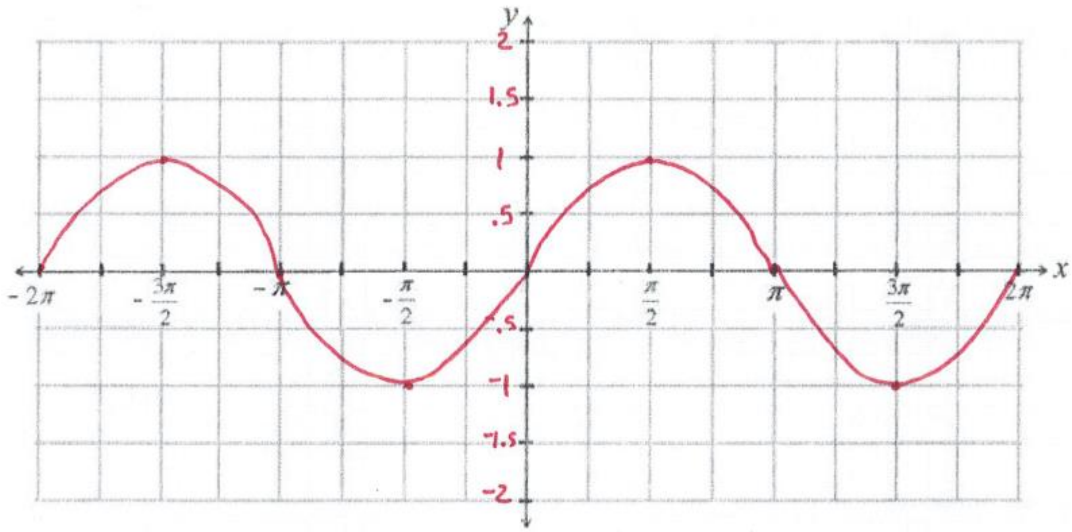
7. Fill out the "Properties" box.

8. Complete all steps again using the cosine function and the tangent functions.

9. NOW!!! The fun begins. Once you've completed the: sine, cosine, and tangent functions; cosecant, secant, and cotangent should be a breeze ;)

sin θ

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
sin(x)	**0**	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	**1**	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	**0**	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	**-1**	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	**0**



Properties of the sine function:

1. Period: 2π
2. Domain: All Real Numbers
3. Range: $-1 \leq y \leq 1$
4. x-intercepts: $\pi(n)$, where n is an integer
5. y-intercept: 0
6. maximum values: 1
7. minimum values: -1

a. When is sin (θ) a positive number?

Quadrants 1 and 2

b. When is sin (θ) a negative number?

Quadrants 3 and 4

c. When is sin (θ) = 0?

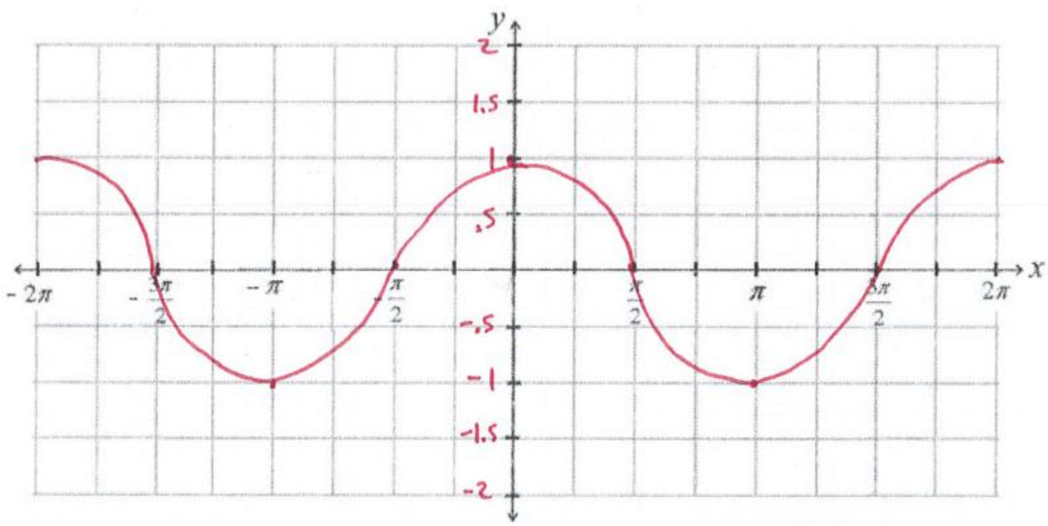
$-2\pi, -\pi, 0, \pi, 2\pi$

d. When is sin (θ) = ± 1?

$-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos\ \theta$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
cos(x)	**1**	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	**0**	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	**-1**	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	**0**	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	**1**



a. When is cos (θ) a positive number?

Quadrants 1 and 4

b. When is cos (θ) a negative number?

Quadrants 2 and 3

c. When is cos (θ) = 0?

$-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

d. When is cos (θ) = ± 1?

$-2\pi, -\pi, 0, \pi, 2\pi$

Properties of the cos function:

1. Period: 2π

2. Domain: All Real Numbers

3. Range: $-1 \leq y \leq 1$

4. x-intercepts: $\frac{\pi}{2} + \pi(n)$, where n is an integer

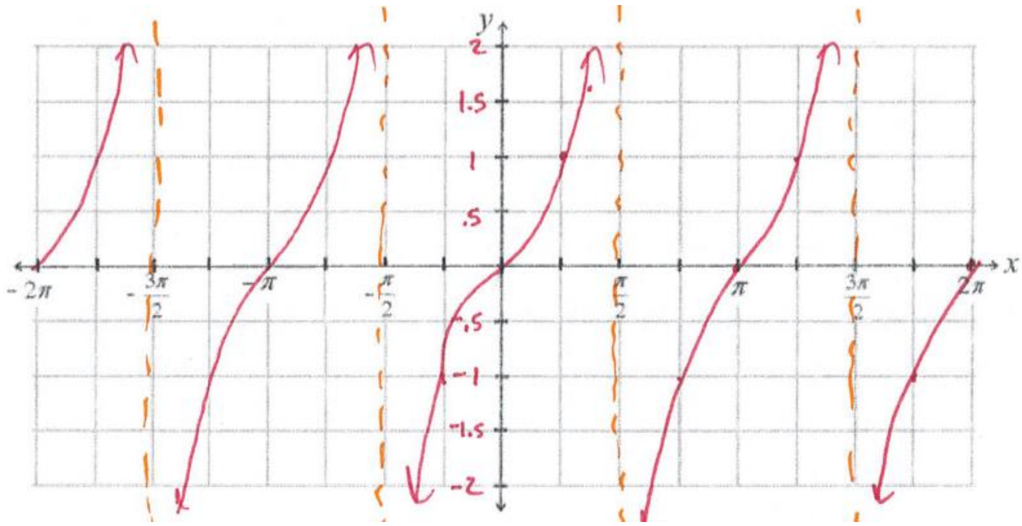
5. y-intercept: 1

6. maximum values: 1

7. minimum values: -1

tan θ

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
tan(x)	**0**	$\frac{\sqrt{3}}{3}$	**1**	$\sqrt{3}$	**Undefined**	$-\sqrt{3}$	**-1**	$-\frac{\sqrt{3}}{3}$	**0**	$\frac{\sqrt{3}}{3}$	**1**	$\sqrt{3}$	**Undefined**	$-\sqrt{3}$	**-1**	$-\frac{\sqrt{3}}{3}$	**0**



a. When is tan (θ) a positive number?

Quadrants 1 and 3

b. When is tan (θ) a negative number?

Quadrants 2 and 4

c. When is tan (θ) = 0?

$-2\pi, -\pi, 0, \pi, 2\pi$

d. When is tan (θ) = ± 1?

$-2\pi, -\pi, 0, \pi, 2\pi$

e. When is tan (θ) undefined?

$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

f. What happens on the graph when tan (θ) is undefined?

Vertical Asymptote

<u>Properties of the tangent function:</u>

1. Period: π

2. Domain: All Real Numbers except for $\frac{\pi}{2}(n)$, where n is odd.

3. Range: All Real Numbers

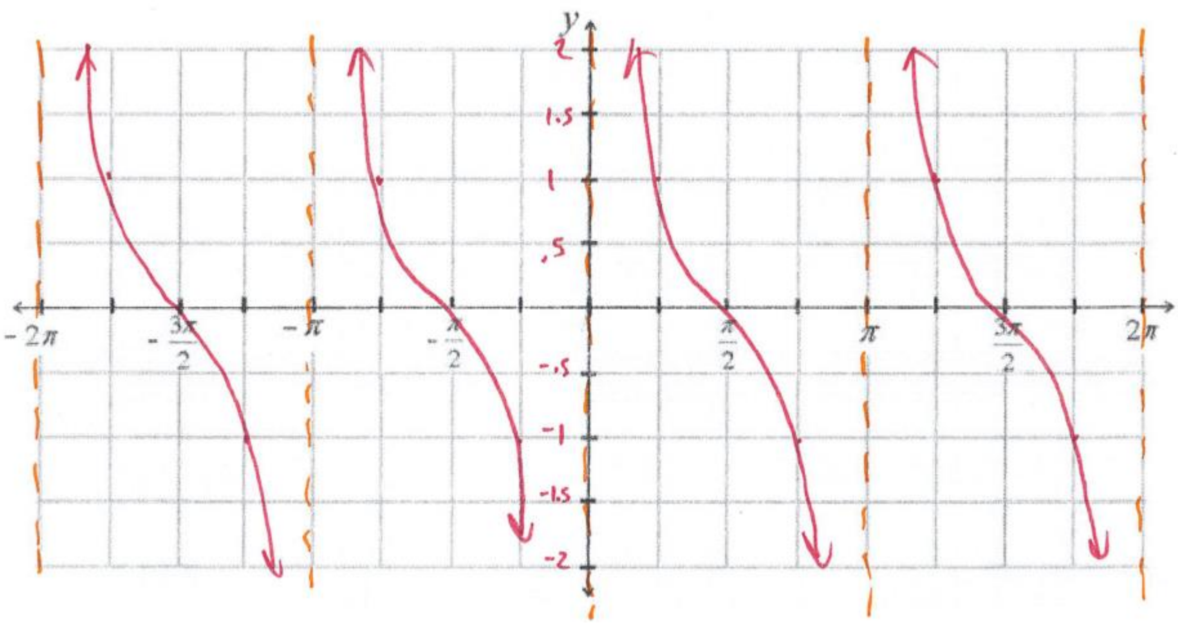
4. x-intercepts: $\pi(n)$, where n is an integer

5. y-intercept: 0

6. asymptotes: $x = \frac{\pi}{2}(n)$, where n is odd.

cot θ

x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
cot(x)	**0**	$\frac{\sqrt{3}}{3}$	**1**	$\sqrt{3}$	**Undefined**	$-\sqrt{3}$	**-1**	$-\frac{\sqrt{3}}{3}$	**0**	$\frac{\sqrt{3}}{3}$	**1**	$\sqrt{3}$	**Undefined**	$-\sqrt{3}$	**-1**	$-\frac{\sqrt{3}}{3}$	**0**



Properties of the cotangent function:

1. Period: π

2. Domain: All Real Numbers except for $\pi(n)$, where n is an integer.

3. Range: All Real Numbers

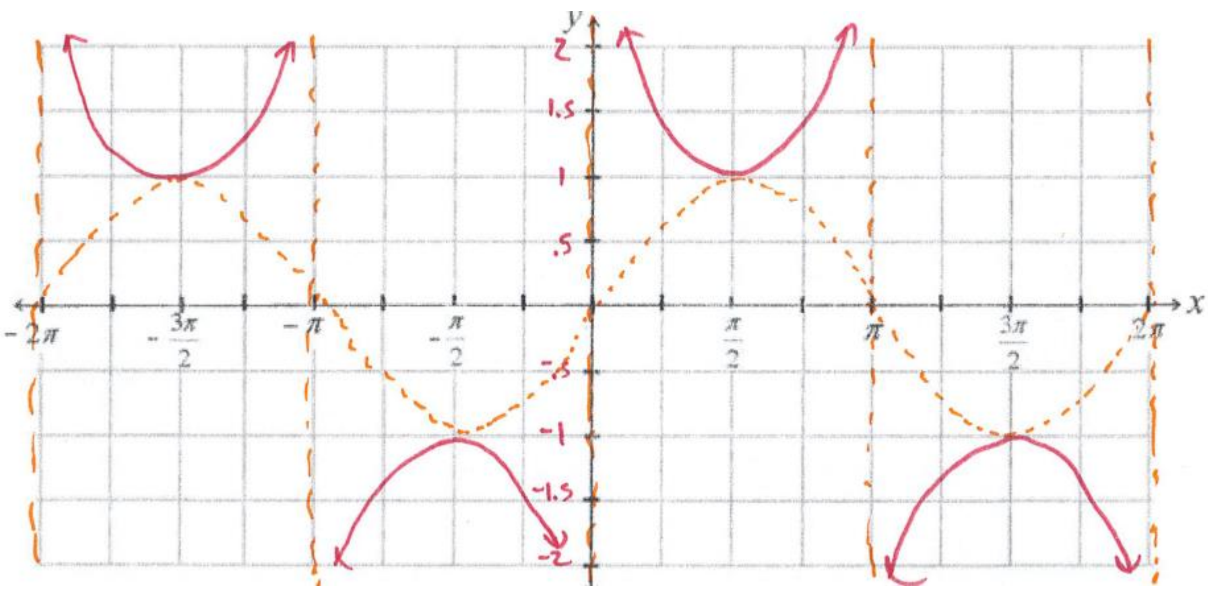
4. x-intercepts: $\frac{\pi}{2}(n)$, where n is odd.

5. y-intercept: none

6. asymptotes: $x = \pi(n)$, where n is an integer.

csc θ

x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
csc(x)	Undefined	$\sqrt{2}$	1	$\sqrt{2}$	Undefined	$-\sqrt{2}$	-1	$-\sqrt{2}$	Undefined	$\sqrt{2}$	1	$\sqrt{2}$	Undefined	$-\sqrt{2}$	-1	$-\sqrt{2}$	Undefined



Remember: $\csc(\theta) = \frac{1}{\sin(\theta)}$

To graph:

1. Graph the related sine function.
2. Graph the asymptotes at the zeros.
3. Graph the cosecant function.

Properties of the cosecant function:

1. Period: 2π

2. Domain: All Real Numbers except for $\pi(n)$, where n is an integer

3. Range: $y \geq 1$ or $y \leq -1$

4. x-intercepts: none

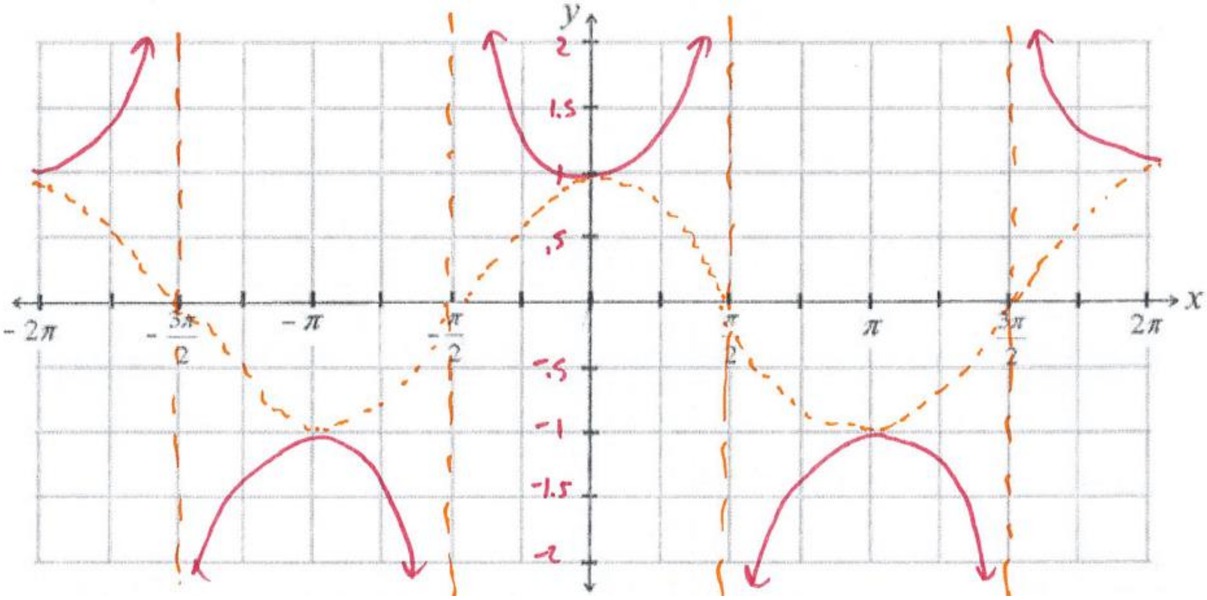
5. y-intercept: none

6. maximum values: $y = -1$ (relative max.)

7. minimum values: $y = 1$ (relative min.)

sec θ

x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
sec(x)	**1**	$\sqrt{2}$	**Undefined**	$-\sqrt{2}$	**-1**	$-\sqrt{2}$	**Undefined**	$\sqrt{2}$	**1**	$\sqrt{2}$	**Undefined**	$-\sqrt{2}$	**-1**	$-\sqrt{2}$	**Undefined**	$\sqrt{2}$	**1**



Remember: $\sec(\theta) = \frac{1}{cos(\theta)}$

To graph:

1. Graph the related cosine function.
2. Graph the asymptotes at the zeros.
3. Graph the secant function.

Properties of the secant function:

1. Period: 2π
2. Domain: All Real Numbers except for $\frac{\pi}{2}(n)$, where n is odd
3. Range: $y \geq 1 \ or \ y \leq -1$
4. x-intercepts: none
5. y-intercept: 1
6. maximum values: $y = -1$ (relative max.)
7. minimum values: $y = 1$ (relative min.)

Still need help? Visit: http://bit.ly/GraphingTrigFunctions

Trig Graphing Packet #2

Chapter 6 – Advanced Math

…..“I like to move it, move it”…..

Learning Targets:

- I can find the amplitude of sine and cosine waves.
- I can find the period of sine and cosine waves.
- I can write equations for sine and cosine waves given the amplitude and period.
- I can find the vertical shift for a sine and cosine wave.
- I can find the phase shift for a sine and cosine wave.
- I can write the equations of sine and cosine waves given the phase shift, and vertical shift.
- I can put it all together and write the equations of sine and cosine waves given the period, amplitude, phase shift and vertical shift!!

Calculator Activity:

Putting it all together

$$y = A\sin\left(B(x \pm C)\right) \pm D$$

Transformation/translation	Definition	Impacts	Action
AMPLITUDE **A**	vertical scale change half the value between the highest and lowest points in the cycle	y-values	|A| - amplitude Multiply by A, including its sign. If negative, flip at end.
VERTICAL SHIFT **D**	translation up or down	y-values	Add or subtract D, which moves the midline.
PHASE SHIFT **C**	translation left or right	x-values	Add or subtract C, which moves the graph left or right. (OPPOSITE SIGN)
PERIOD **B**	horizontal scale change Normal period multiplied by $\frac{1}{\|B\|}$ equals the new period.	x-values	The normal period for sine and cosine is 2π and the normal period for tangent is π.

Order for Graphing Sine and Cosine functions:

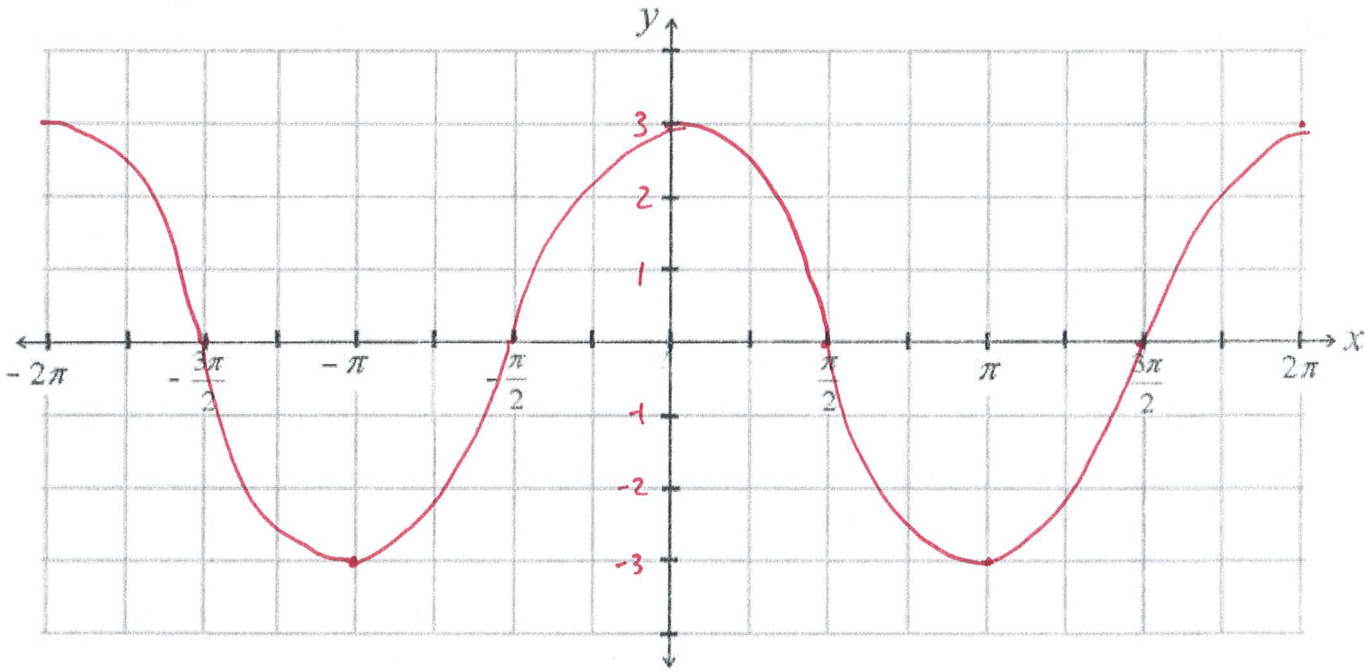
1. B - Period

2. A – Amplitude (flip while plotting if negative)

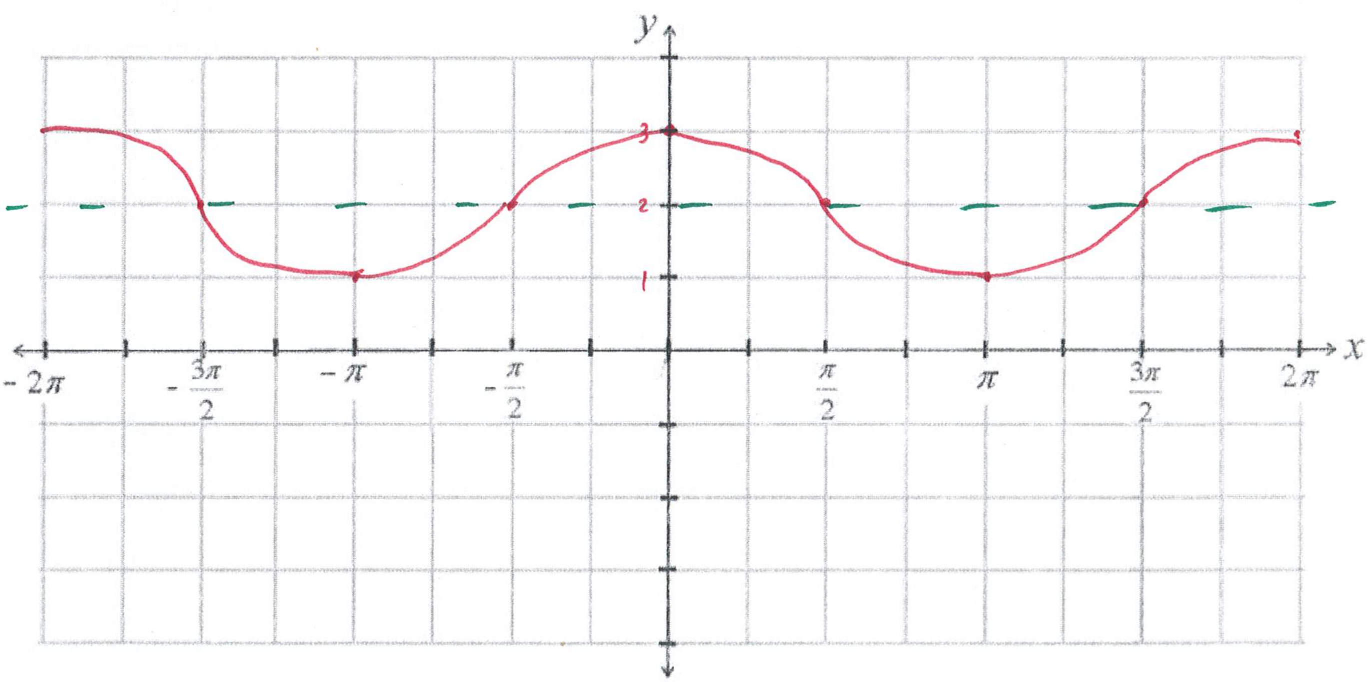
3. D – Vertical Shift

4. C – Phase Shift

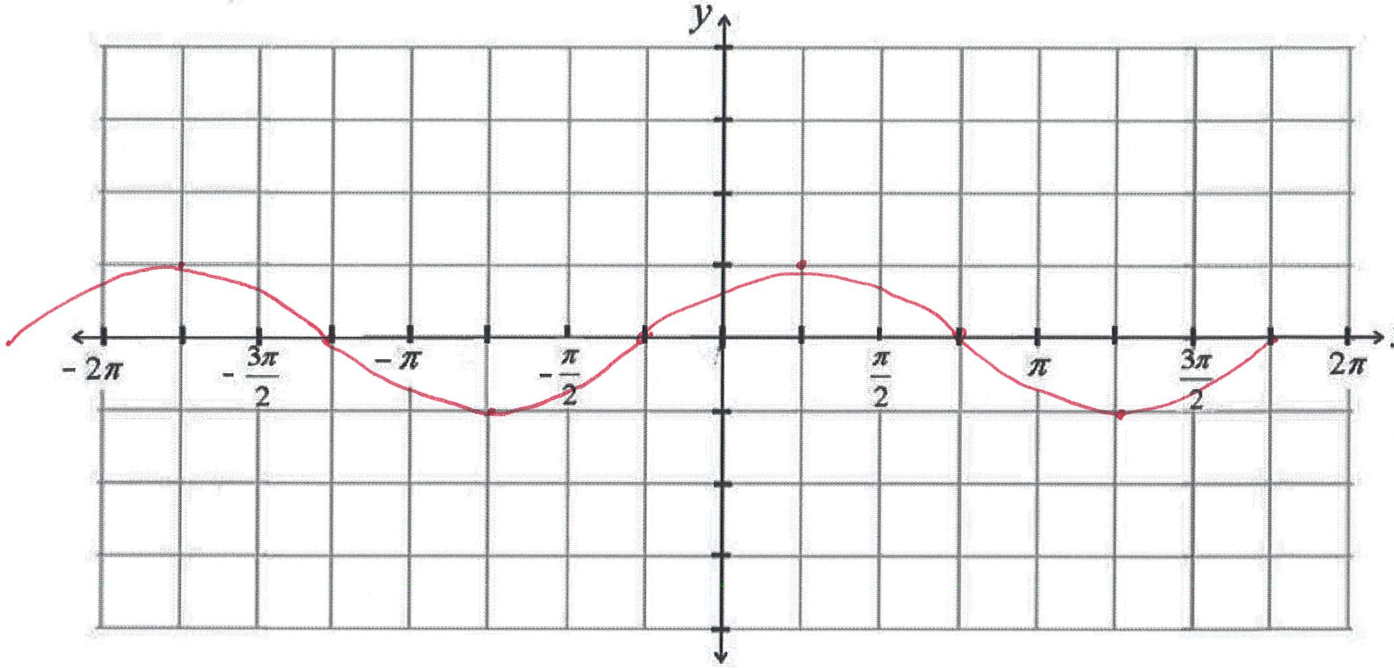
Amplitude: EXAMPLE 1: **$y = 3\cos x$**



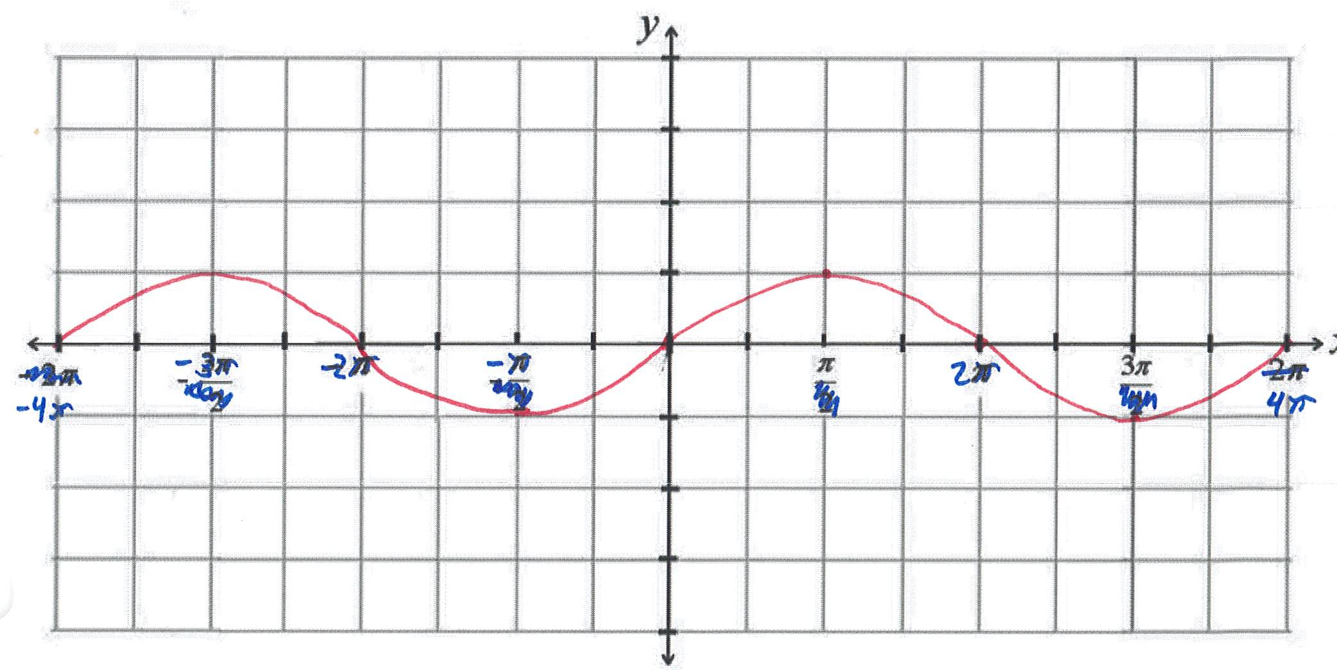
VERTICAL SHIFT: Example 2: **$y = \cos x + 2$**



PHASE SHIFT: Example 3. $y = \sin\left(x + \frac{\pi}{4}\right)$



PERIOD: Example 4. $y = \sin\left(\frac{x}{2}\right)$ $y = \sin\left(\frac{1}{2}x\right)$ $\frac{2\pi}{\frac{1}{2}} = 4\pi$



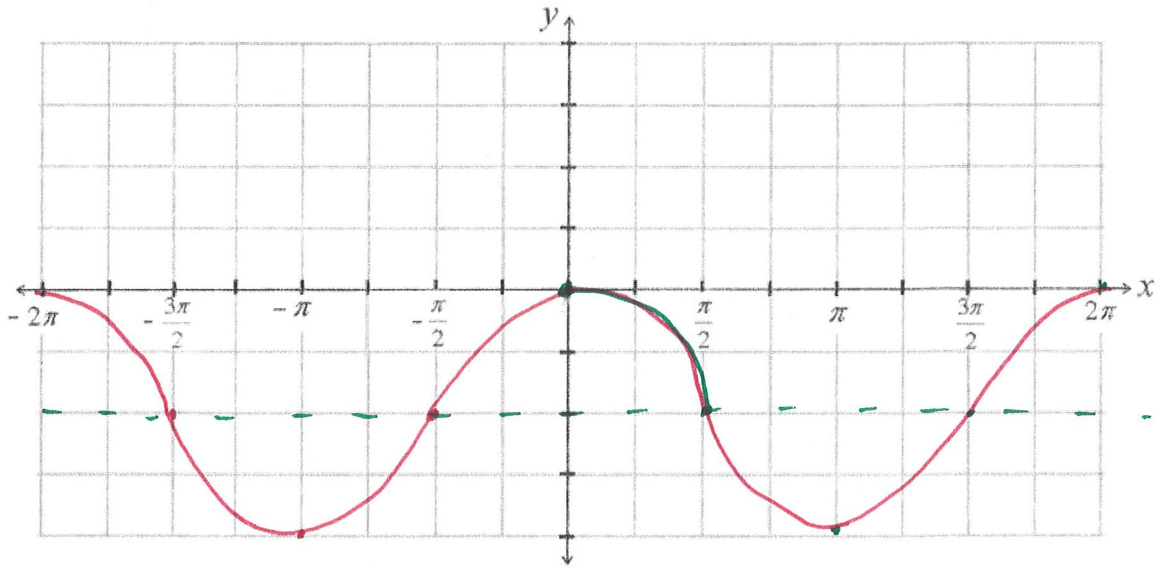
Example 5: Graph: $y = 2\cos x - 2$

Amplitude: 2

Period: 2π

Phase shift: —

Vertical Shift: Down 2



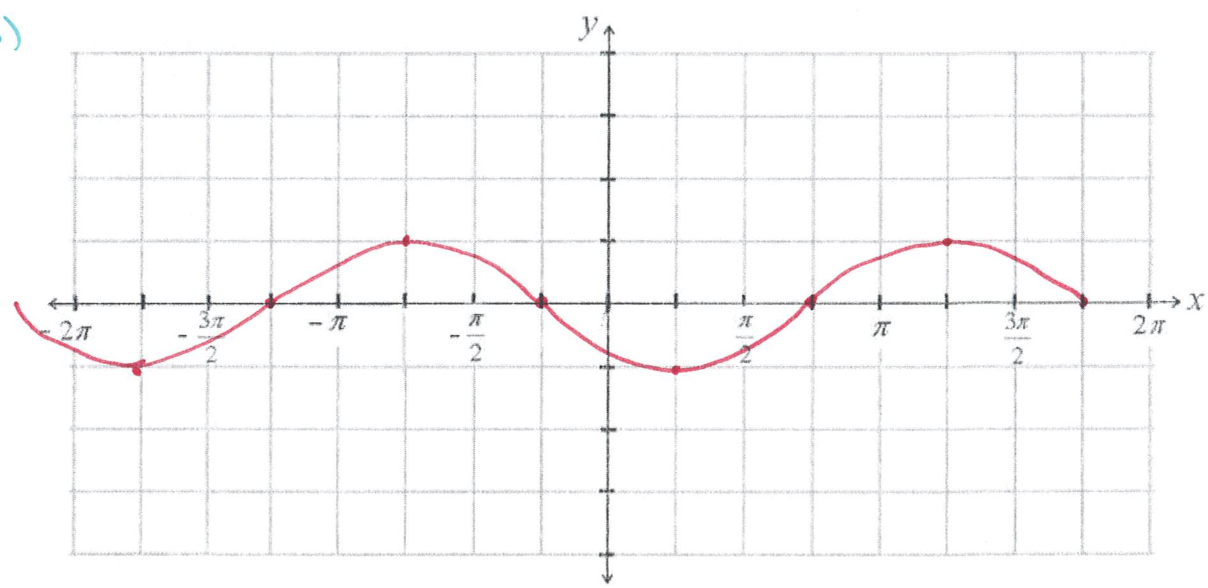
EXAMPLE 6: Graph $y = -\sin\left(x + \frac{\pi}{4}\right)$

Amplitude: 1 (Flip)

Period: 2π

Phase shift: $-\frac{\pi}{4}$

Vertical Shift: —



EXAMPLE 7: Write an equation of the cosine function with amplitude 9.8 and period 6π.

$\frac{2\pi}{B} = 6\pi$

$\frac{2\pi}{6\pi} = B$

$B = \frac{1}{3}$

$y = 9.8\cos\left(\frac{x}{3}\right)$

$y = 4\cos\left(\frac{1}{2}(x + 2\pi)\right) - 6$

EXAMPLE 8: State the amp, period, ph. shift, and v. shift of $y = 4\cos\left(\frac{x}{2} + \pi\right) - 6.$ Then graph it.

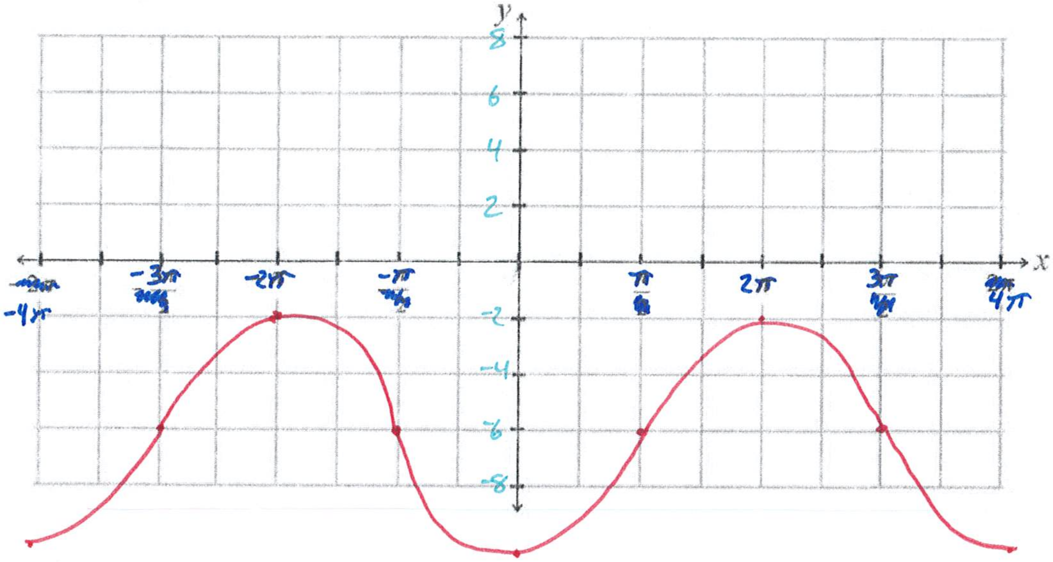
Amplitude: 4

Period: 4π

$\frac{2\pi}{\frac{1}{2}}$

Phase shift: -2π

Vertical Shift: -6



Example 9: Write and equation of a sine function with amplitude 4, period π, phase shift $\frac{-\pi}{8}$ and vertical shift 6, then graph it.

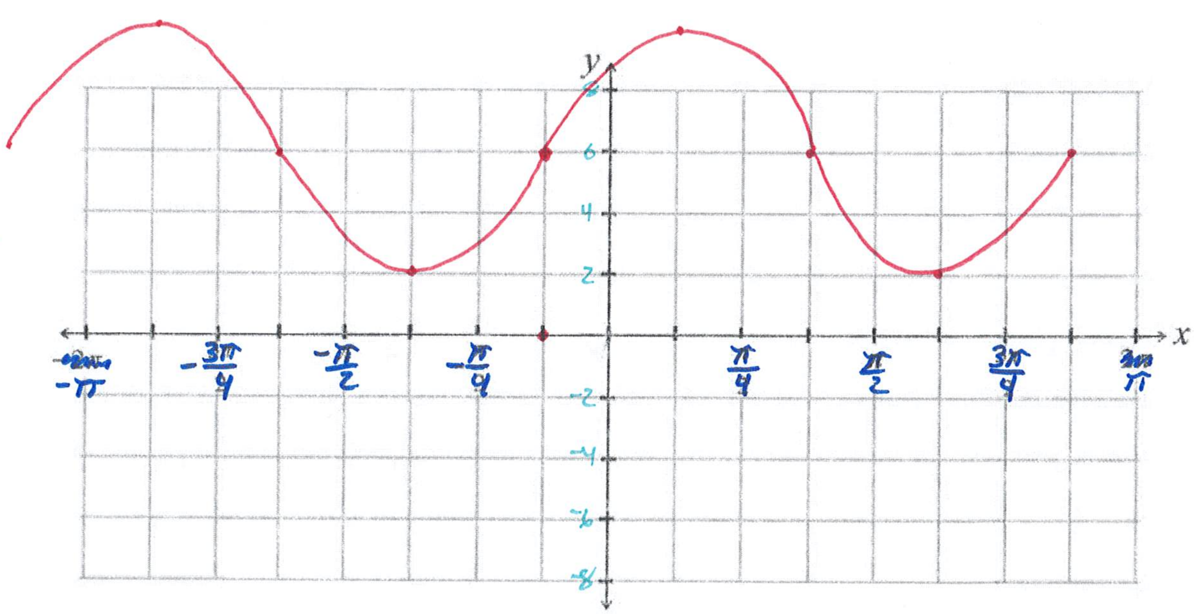
Equation: $y = 4\sin\left(2\left(x + \frac{\pi}{8}\right)\right) + 6$

Amplitude: 4

Period: π

Phase shift: $-\frac{\pi}{8}$

Vertical Shift: 6



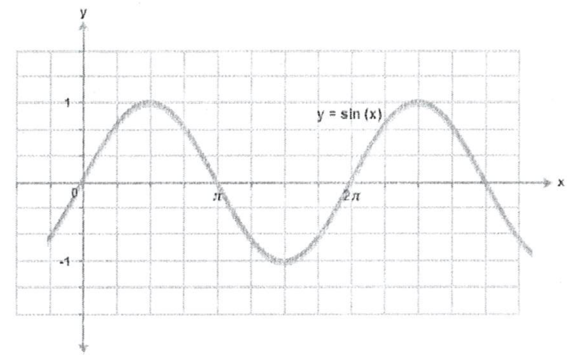
TRIG GRAPHING PACKET #3

Chapter 6 – Advanced Math

Objectives:

- Graph inverse trigonometric functions.
- Find principal values of inverse trig functions.

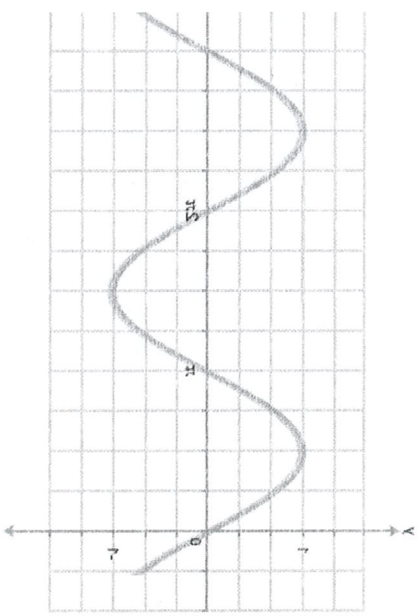
sin x



REMEMBER:

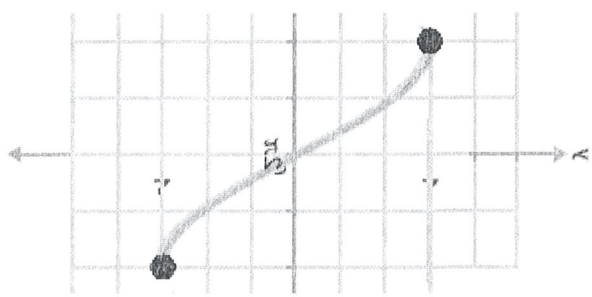
To graph any inverse, reflect the original graph over the line y = x. (Which is also the same thing as flipping the x and y coordinates).

sin $^{-1}$ ***x***



NOTE:

This is, obviously, not a function, so we crop the picture in order to image it one. Typically, we do so like this:



The principal values for inverse sin x, are these range values. The answers that your calculator will give you will always be in this range. It is up to YOU to accommodate for the correct quadrant

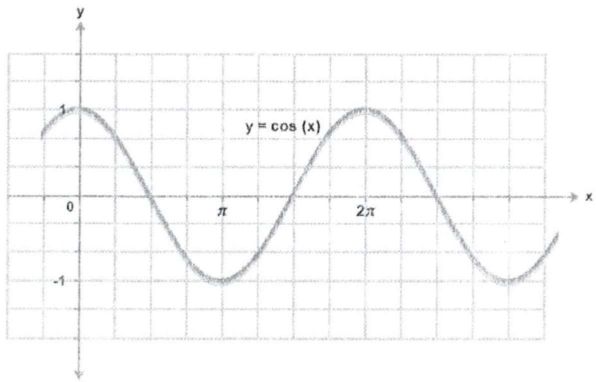
Domain: -1 < x < 1

Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

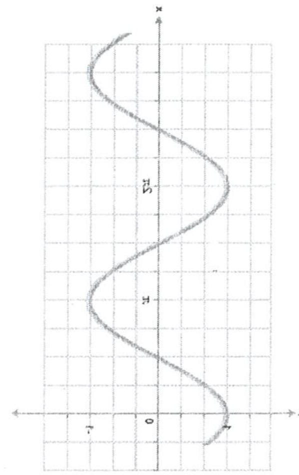
Which QUADRANT?

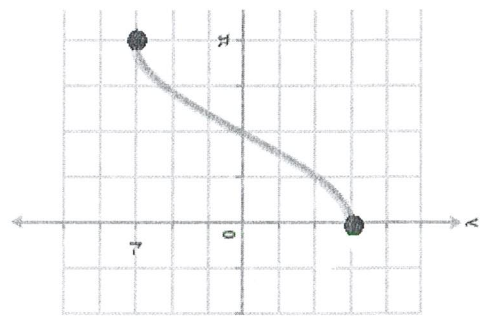
Q I and Q IV

cos x



cos* $^{-1}$ *x





The *principal values* for inverse cos x, are again the range values.

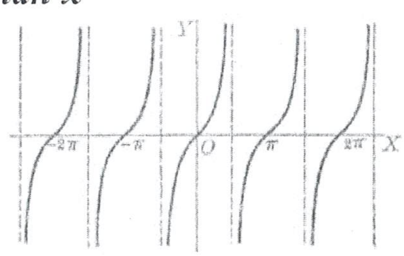
Domain: -1 < x < 1

Range: $0 \le y \le \pi$

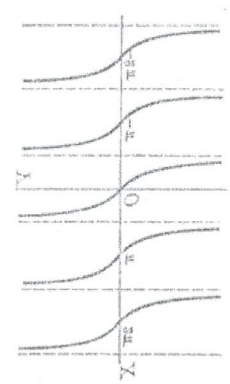
Which QUADRANT?

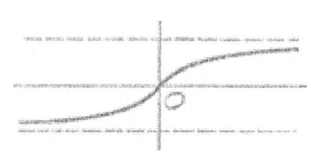
Q I and Q II

tan x



tan* $^{-1}$ *x





The principal values for inverse tan x are the same as inverse sin x.

Domain: all real

Range: $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

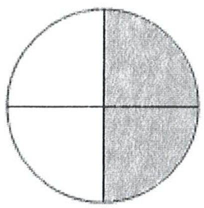
Which QUADRANT?

Q I and Q IV

Principal Values *(Range values from inverse functions)*

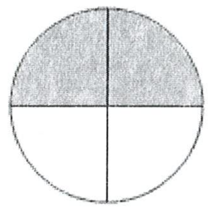
Inverse *sin x*

$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



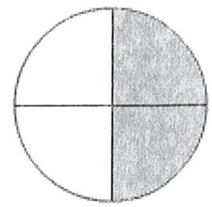
Inverse *cos x*

$0 \leq y \leq \pi$



Inverse *tan x*

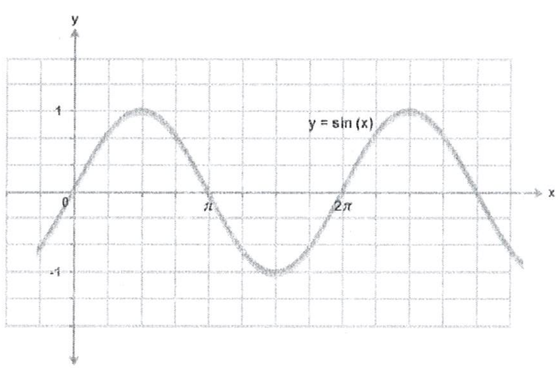
$-\frac{\pi}{2} < y < \frac{\pi}{2}$



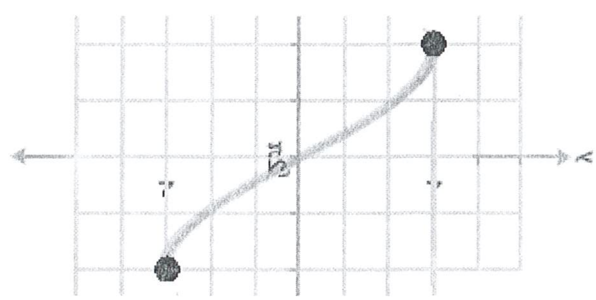
Notice that the quadrants correspond to where sin, cos, and tan are both positive and negative

RECAP – Trig Inverses and Their Graphs

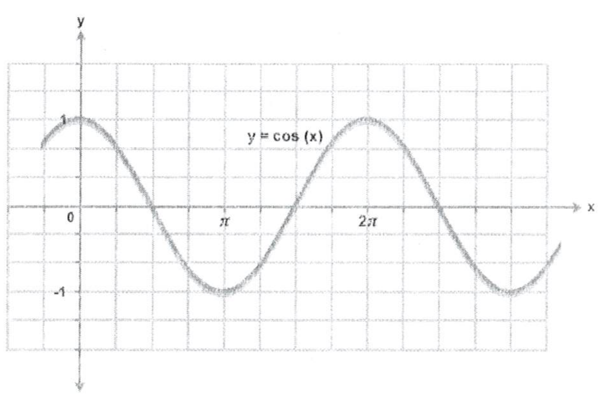
sin x



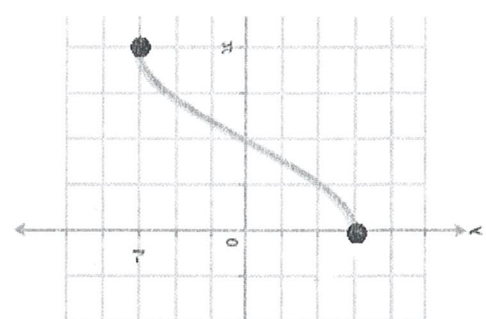
sin $^{-1}$ ***x***



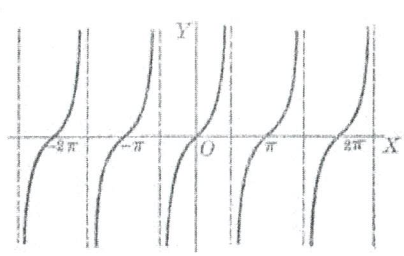
cos x



cos $^{-1}$ ***x***



tan x



tan $^{-1}$ ***x***

Example1:

Write the inverse of the equation $y = \tan^{-1} 2x$.

$x = \tan^{-1}(2y)$

$\tan x = 2y$

$y = \frac{\tan x}{2}$

Example 2 (think back to Chapter 5 as well): **Principal Values ONLY!**

a. $\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)$

$\frac{7\pi}{4} \pm 2\pi n$

b. $\sin^{-1}\left(\cos\frac{\pi}{2}\right)$

$\sin^{-1}(0)$

$\pm \pi n$

c. $\sin\left(\tan^{-1} 1 - \sin^{-1} 1\right)$

$\sin\left(\frac{\pi}{4} - \frac{\pi}{2}\right)$

$\sin\left(-\frac{\pi}{4}\right)$

$-\frac{\sqrt{2}}{2}$

Your Turn : Solve

a. arcsin (-1)

$\frac{3\pi}{2}$

b. $\sin^{-1}(\cos 2\pi)$

$\sin^{-1}(1)$

$\frac{\pi}{2}$

c. $\sin(\tan^{-1} 1 - \sin^{-1} 0)$

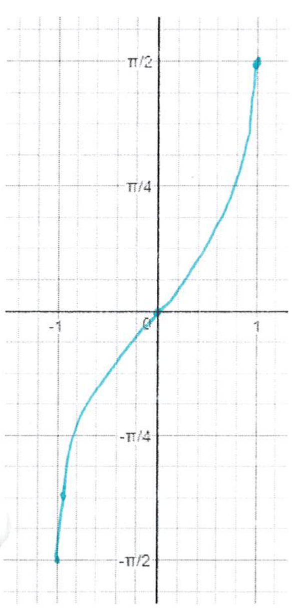
$\sin\left(\frac{\pi}{4} - 0\right)$

$\sin\left(\frac{\pi}{4}\right)$

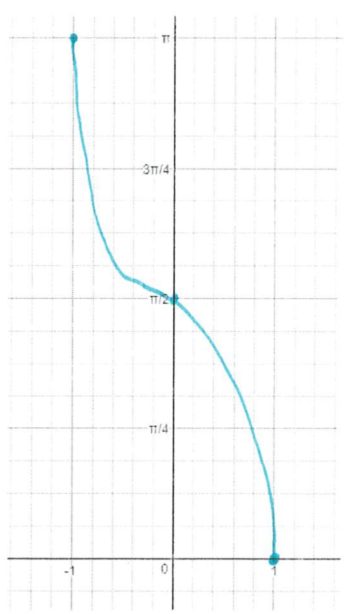
$\frac{\sqrt{2}}{2}$

Graph :

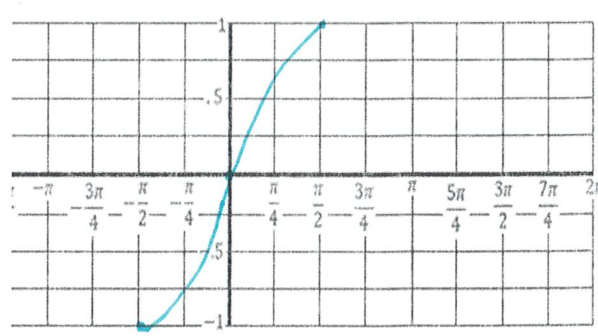
Graph $\sin^{-1}(x)$



Graph $\cos^{-1}(x)$



Graph $\tan^{-1}(x)$



TRIG GRAPHING PACKET #4

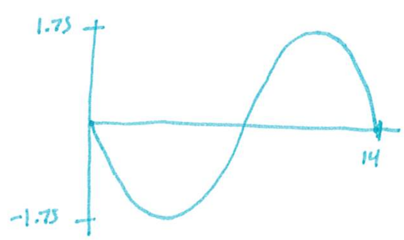
Objectives:

- Use trig function graphing knowledge to solve word problems.

Example 1

A signal buoy between the coast of Hilton Head Island, South Carolina, and Savannah, Georgia, bobs up and down in a minor squall. From the highest point to the lowest point, the buoy moves a distance of $3\frac{1}{2}$ feet. It moves from its highest point down to its lowest point and back to its highest point every 14 seconds. Find an equation of the motion for the buoy assuming that it is at its equilibrium point at $t = 0$ and the buoy is on its way down at that time. What is the height of the buoy at 8 seconds and at 17 seconds?

Let's sketch and label:



a. Find an equation for the motion of the buoy.

$A = \frac{3.5}{-2} = -1.75$

$D = 0$

$B = \frac{2\pi}{B} = 14 = \frac{2\pi}{14} = \frac{\pi}{7}$

$y = -1.75 \sin\left(\frac{\pi}{7}x\right)$

b. Determine the height of the buoy at 8 seconds and at 17 seconds.

.759 ft.   -1.706 ft.

REMEMBER:

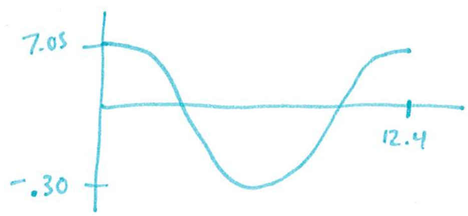
Amplitude is half the value between the highest and lowest points in the cycle.

> **NOTE:**
> In general, any sinusoidal function can be written as a sine function of as a cosine function. The amplitude, the period, and the midline will remain the same. However, the phase shift will be different. To avoid a greater phase shift than necessary, you may wish to use the sine function if the function is about zero at $x = 0$ and a cosine function if the function is about the maximum or minimum at $x = 0$.

Example 2

One day in March in Hilton Head, South Carolina, the first high tide occurred at 6:18 A.M. The high tide was 7.05 feet, and the low tide was -0.30 feet. The period for the oscillation of the tides is 12 hours and 24 minutes.

Let's sketch and label:



$A = \frac{7.05 - (-.3)}{2} = 3.675$

$B: \frac{2\pi}{12.4}$

$C: 6.3$

$D = \frac{7.05 + (-.3)}{2} = 3.375$

a. Determine what time the next high tide will occur.

6:42 PM

b. Write the period of the oscillation as a decimal.

$\approx .5067$

c. What is the amplitude of the sinusoidal function that models the tide?

3.675

d. If $t = 0$ represents midnight, write a sinusoidal function that models the tide.

$y = 3.675 \cos\left(\frac{2\pi}{12.4}(x - 6.3)\right) + 3.375$

e. At what time will the tides be 6 feet for the first time that day?

4:46 A.M.

Example 3

A leaf floats on the water bobbing up and down. The distance between its highest and lowest point is 4 centimeters. It moves from its highest point down to its lowest point and back to its highest point every 10 seconds. Write a <u>cosine function</u> that models the movement of the leaf in relationship to the equilibrium point.

$A = 2$

$B = \frac{2\pi}{10} = \frac{\pi}{5}$

$$y = 2\cos\left(\frac{\pi}{5}x\right)$$

Example 3

Write a <u>sine function</u> which models the oscillation of tides in Savannah, Georgia, if the equilibrium point is 4.24 feet, the amplitude is 3.55 feet, the phase shift is -4.68 hours, and the period is 12.40 hours.

$A = 3.55$

$B = \frac{2\pi}{12.4}$

$C = -4.68$

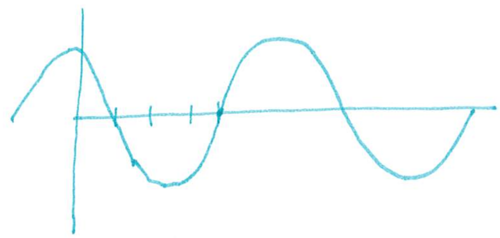
$D = 4.24$

$$y = 3.55\sin\left(\frac{5\pi}{31}(x+4.68)\right) + 4.24$$

Example 4

The mean average temperature in Buffalo, New York, is 47.5°. The temperature fluctuates 23.5° above and below the mean temperature. If $t = 1$ represents January, the phase shift of the sine function is 4.

Let's sketch and label:



a. Write a model for the average monthly temperature in Buffalo. (ie. EQUATION!)

$A = 23.5$

$B = \frac{\pi}{6}$

$C = 4$

$D = 47.5$

$$y = 23.5 \sin\left(\frac{\pi}{6}(x-4)\right) + 47.5$$

b. According to your model, what is the average temperature in March?

35.75°

c. According to your model, what is the average temperature in August?

67.9°