Chapter 2 – Matrix Operations and Applications Assignments Name Key

SHOW ALL WORK ON A SEPARATE SHEET OF PAPER!

Section 2.1:

1. Using your knowledge of matrices, answer the following:
   a. How many elements does a 2 x 8 matrix have? 16
   b. How many elements does a 1 x 7 matrix have? 7
   c. How many elements does an *m* x *n* matrix have? mn

2. A trendy garment shop receives orders from three clothing companies. The first shop orders 25 jackets, 75 shirts, and 75 pairs of pants. The second shop orders 30 jackets, 50 shirts, and 50 pairs of pants. The third shop orders 20 jackets, 40 shirts, and 35 pairs of pants. Display this information in a matrix. Let the rows represent the shops and the columns represent the type of garment ordered. Label the rows and columns of your matrix accordingly.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} J & S & P \end{array} \\ \begin{bmatrix} 25 & 75 & 75 \\ 30 & 50 & 50 \\ 20 & 40 & 35 \end{bmatrix} \end{array}$$

3. Given the following matrix, answer the questions below:

$$A = \begin{bmatrix} 21 & 14 & 31 \\ 17 & 12 & 5 \\ 8 & 72 & 32 \end{bmatrix}$$

   a. What is the value of A_{21}, A_{12}, and A_{32}? 17, 14, 72
   b. If the rows represented shirts, shoes, and pants and the columns represented TJMaxx, JCPenny, and Kohl's, what is the interpretation for entry A_{21}?

   17 shoe orders from TJMaxx.

4. If A = $\begin{bmatrix} 10 & -5 & 13 \\ -7 & 14 & 9 \end{bmatrix}$ and B = $\begin{bmatrix} -6 & 2 & -8 \\ -4 & 5 & -3 \end{bmatrix}$, find A + B and B + A. Is matrix addition **commutative**? (Does A + B = B + A?)

$$A+B = \begin{bmatrix} 4 & -3 & 5 \\ -11 & 19 & 6 \end{bmatrix}$$

$$B+A = \begin{bmatrix} 4 & -3 & 5 \\ -11 & 19 & 6 \end{bmatrix}$$

Yes, matrix addition is commutative.

5. If A = $\begin{bmatrix} 10 & -5 & 13 \\ -7 & 14 & 9 \end{bmatrix}$ and B = $\begin{bmatrix} -6 & 2 & -8 \\ -4 & 5 & -3 \end{bmatrix}$, find A – B and B – A. Is matrix subtraction **commutative?** (Does A - B = B - A?)

No, matrix subtraction is not commutative.

$A-B=\begin{bmatrix} 16 & -7 & 21 \\ -3 & 9 & 12 \end{bmatrix}$ $B-A=\begin{bmatrix} -16 & 7 & -21 \\ 3 & -9 & -12 \end{bmatrix}$

6. For breakfast, Bobbi had cereal, a banana, a cup of milk, and a slice of toast. She recorded the following information in her food journal. Cereal: 165 calories, 3 g fat, 33 g carbs, and no cholesterol. Banana: 120 calories, no fat, 26 g carbs, and no cholesterol. Milk: 120 calories, 5 g fat, 11 g carbs, and 15 mg cholesterol. Toast: 125 calories, 6 g fat, 14 g carbs, and 18 mg cholesterol.

   a. Write the information in a matrix *N* whose rows represent the foods. Label the rows and columns of your matrix.

$$\begin{array}{c} \\ C \\ B \\ M \\ T \end{array} \begin{array}{c} \begin{array}{cccc} \text{Cal.} & \text{fat} & \text{car.} & \text{cho.} \end{array} \\ \begin{bmatrix} 165 & 3 & 33 & 0 \\ 120 & 0 & 26 & 0 \\ 120 & 5 & 11 & 15 \\ 125 & 6 & 14 & 18 \end{bmatrix} \end{array}$$

   b. State the values of N_{23}, N_{32}, and N_{41}. 26, 5, 125

   c. Write an interpretation of N_{23}. 26 carbs in a banana

7. Find the value of each of the following expressions or explain why it is not possible.

   a. $\begin{bmatrix} 2 & 4 \\ -3 & 5 \end{bmatrix}+\begin{bmatrix} 7 & -8 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9 & -4 \\ -4 & 7 \end{bmatrix}$

   b. $\begin{bmatrix} 1 & 5 & 7 \\ 0 & 2 & 4 \end{bmatrix}+\begin{bmatrix} 1 & 0 \\ 5 & 2 \\ 7 & 4 \end{bmatrix}$ Not Possible

   c. $\begin{bmatrix} 1 & 5 & 10 \end{bmatrix}-\begin{bmatrix} 1 \\ 5 \\ 10 \end{bmatrix}$ Not Possible

   d. $\begin{bmatrix} 1 & 0 \\ 3 & 0 \\ 5 & 0 \\ 0 & -4 \end{bmatrix}-\begin{bmatrix} 1 & 5 \\ -3 & 0 \\ 2 & -4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ 6 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix}$

   e. $\begin{bmatrix} -2y \\ x \end{bmatrix}=\begin{bmatrix} 4-5x \\ y+5 \end{bmatrix}$ (-2, -7)

   f. $\begin{bmatrix} 4x-1 \\ 9y+5 \end{bmatrix}=\begin{bmatrix} 3x \\ y-3 \end{bmatrix}$ (1, -1)

$-2y = 4-5x$
$x = y+5$

$-2y = 4-5(y+5)$
$-2y = 4-5y-25$
$3y = -21$
$y = -7$

$x = -7+5$
$x = -2$

$4x-1 = 3x$
$x = 1$

$9y+5 = y-3$
$y = -1$

8. The matrix M below shows the mileage between 5 major US cities.

$$M = \begin{array}{cccccc} & Atlanta & Boston & Chicago & LA & St.Louis \\ Atlanta & 0 & 1075 & 716 & 2211 & 555 \\ Boston & 1075 & 0 & 1015 & 3026 & 1187 \\ Chicago & 716 & 1015 & 0 & 2034 & 297 \\ LA & 2211 & 3026 & 2034 & 0 & 1842 \\ St.Louis & 555 & 1187 & 297 & 1842 & 0 \end{array}$$

a. Entries that are located in row *i*, column *j*, where *i* = *j*, are said to be located on the **main diagonal** of the matrix. Examine the entries on the main diagonal of *M*. What do you notice? All zeros.

b. A square matrix *R* with order *n x n* is **symmetric** if $R_{ij} = R_{ji}$. Is matrix *M* symmetric? Explain. Yes, it is symmetric over the diagonal.

c. Give an example of a 3 x 3 matrix that is symmetric.

$$\begin{bmatrix} 0 & 4 & 3 \\ 4 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}$$

d. Could a matrix that is not square ever be symmetric? Why?

No because the dimensions would not match.

Section 2.2:

1. The regents at a state university recently announced a 7% raise of tuition rates per semester hour. The current rates per semester hour are shown in the following table:

	Undergraduate	Graduate
Resident	\$75.00	\$99.25
Non-resident	\$204.00	\$245.25

$$C = \begin{array}{c} \\ R \\ N \end{array}\begin{array}{c} \begin{array}{cc} U & \quad G \end{array} \\ \begin{bmatrix} 75 & 99.25 \\ 204 & 245.25 \end{bmatrix} \end{array}$$

   a. Write and label a matrix that represents the table above.

   b. Find a new matrix that represents the tuition rates per semester hour after the 7% raise goes into effect. Label your matrix.

$$1.07C = \begin{array}{c} \\ R \\ N \end{array}\begin{array}{c} \begin{array}{cc} U & \quad G \end{array} \\ \begin{bmatrix} 80.25 & 106.20 \\ 218.28 & 262.42 \end{bmatrix} \end{array} = A$$

   c. Find a matrix that represents the dollar increase for each of the categories. Label your matrix.

$$C - A = \begin{array}{c} \\ R \\ N \end{array}\begin{array}{c} \begin{array}{cc} U & \quad G \end{array} \\ \begin{bmatrix} 5.25 & 6.95 \\ 14.28 & 17.17 \end{bmatrix} \end{array}$$

2. For each of the following state whether the matrix product QP is defined. If so, give the order (dimensions) of the product.
   a. Order of Q: 1 x 4. Order of P: 4 x 2 yes 1×2
   b. Order of Q: 1 x 3. Order of P: 2 x 3 No
   c. Order of Q: 1 x 5. Order of P: 5 x 4 yes 1×4
   d. Order of Q: 1 x 2. Order of P: 4 x 2 No
   e. Order of Q: 1 x *m*. Order of P: *m* x 1 yes 1×1

3. Use the following matrices to compute the given expressions.

$$E = \begin{bmatrix} 3 & 8 & -1 \\ 2 & 0 & 4 \end{bmatrix}, F = \begin{bmatrix} 2 & 4 \end{bmatrix}, \text{ and } G = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

   a. 3*E*

$$\begin{bmatrix} 9 & 24 & -3 \\ 6 & 0 & 12 \end{bmatrix}$$

   d. *FE*

$$\begin{bmatrix} 14 & 16 & 14 \end{bmatrix}$$

   b. *FG*

$$\begin{bmatrix} 20 \end{bmatrix}$$

   e. *EF*

Not Possible

   c. -2*G*

$$\begin{bmatrix} -4 \\ -8 \end{bmatrix}$$

4. A hobby shop has three different locations: North, South, and East. The store's sales for July are shown in the following table.

	North	South	East
Model trains	10	8	12
Model cars	6	5	4
Model planes	3	2	2
Model trucks	4	3	2

Suppose that model trains sell for $40 each, cars for $35, planes for $80, and trucks for $45. Use matrix multiplication to find the shop's total sales at each location. Label your matrices.

$$\begin{array}{cccc} Trn & car & pln & trk \end{array}$$
$$\begin{bmatrix} 40 & 35 & 80 & 45 \end{bmatrix} \cdot \begin{array}{c} \\ trn \\ car \\ pln \\ trk \end{array}\begin{array}{c} \begin{array}{ccc} N & S & E \end{array} \\ \begin{bmatrix} 10 & 8 & 12 \\ 6 & 5 & 4 \\ 3 & 2 & 2 \\ 4 & 3 & 2 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} N & S & E \end{array} \\ \begin{bmatrix} \$1030 & \$790 & \$870 \end{bmatrix} \end{array}$$

5. Using the definitions of matrix multiplication, answer the following:
   a. What must be true about the dimensions of matrices *A* and *B* if the product *C* = *AB* is defined? The number of columns in A must match the number of rows in B.
   b. If the products *AB* and *BA* are both defined, what must be true about the dimensions of matrices *A* and *B*? Why?

      Both need to be square matrices so dimensions match.
   c. Find two nonsquare matrices *A* and *B*, where *AB* and *BA* are both defined. Compute *AB* and *BA*. Are they equal? Why?

$A = \begin{bmatrix} 1 & 2 \end{bmatrix}$ $\quad AB = \begin{bmatrix} 4 \end{bmatrix}$ $\quad$ Dimensions change

$B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\quad BA = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$ $\quad AB \neq BA$

6. Given the matrices A, B, and C:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 1 \\ 2 & 2 \\ -1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

   a. Do you think that A(BC) = (AB)C? Yes
   b. Test your conjecture by computing the products A(BC) and (AB)C.
   c. The computations in part b show one case in which **matrix multiplication is associative**. Do you think this property holds for all matrices A, B, and C for which the product A(BC) is defined? Why or why not?

$BC = \begin{bmatrix} 5 & -2 & 1 \\ 6 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ $\qquad AB = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$

Yes because the order of multiplication remains the same. Commutative properties don't apply!

$$A(BC) = \begin{bmatrix} 6 & 0 & 2 \\ 7 & 2 & 3 \end{bmatrix} = (AB)C = \begin{bmatrix} 6 & 0 & 2 \\ 7 & 2 & 3 \end{bmatrix}$$

Section 2.3:

1. Recall the definition of the **transpose** of matrix *A*.
   a. Describe the transpose of a row matrix.

      Column matrix

   b. Describe the transpose of a column matrix.

      row matrix

   c. Write the transpose of matrix E (E^T) if E = $\begin{bmatrix} 3 & 8 & -1 \\ 2 & 0 & 4 \end{bmatrix}$. $E^T = \begin{bmatrix} 3 & 2 \\ 8 & 0 \\ -1 & 4 \end{bmatrix}$

2. Using matrix A and B below:
   a. Verify that the matrices A and B are inverses of each other by computing AB and BA.

      A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

      $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

      $BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

      $AB = BA = I$, so A and B are inverses.

   b. Not all square matrices will have an inverse. Use algebra to show that matrix C does not have an inverse.

      C = $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

      $\det(C) = 2(6) - 4(3) = 0$

      Inverse does not exist when det = 0.

   c. If you multiply the diagonals of matrix C and subtract their products, what do you get?   0

3. Determine whether each pair of matrices are inverses or not.

   a. $X = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

      Yes!

   c. $X = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}, Y = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

      Yes!

   b. $M = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}, N = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$

      No!

4. Find the inverse of each matrix, if it exists.

a. $\begin{bmatrix} 0 & 2 \\ 4 & 0 \end{bmatrix}$ $\frac{1}{-8}\begin{bmatrix} 0 & -2 \\ -4 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1/4 \\ 1/2 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & -1 \\ 3 & 3 \end{bmatrix}$ $\frac{1}{6}\begin{bmatrix} 3 & +1 \\ -3 & 1 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} & 1/6 \\ -1/2 & 1/6 \end{bmatrix}$

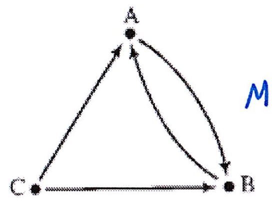
d. $\begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$

Not Possible, inverse does not exist.

Section 2.4:

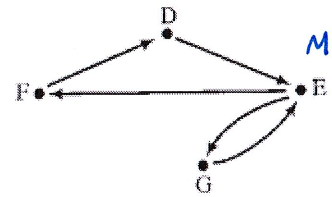
Write the communication matrix for each communication network given below:

1.



$$M = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

2.

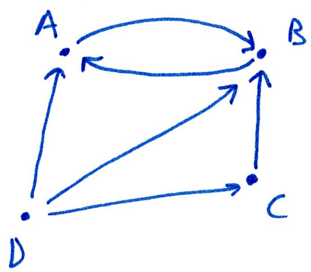


$$M = \begin{array}{c} \\ D \\ E \\ F \\ G \end{array} \begin{array}{c} \begin{array}{cccc} D & E & F & G \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Draw a communication network that is described by the given matrices below:

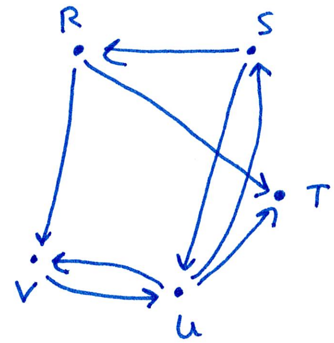
3.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



4.

$$\begin{array}{c} \\ R \\ S \\ T \\ U \\ V \end{array} \begin{array}{c} \begin{array}{ccccc} R & S & T & U & V \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



5. Answer the questions about a shipping scenario below:

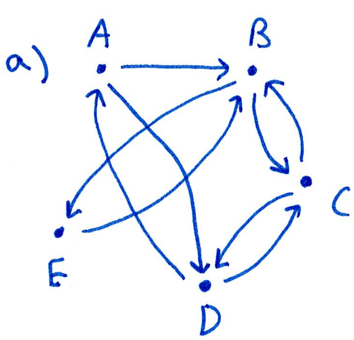
a) Ship A sends messages to Ship B. Ship B sends and receives messages from Ships C and E. Ship D sends and receives messages from Ships A and C. Draw this network below.

b) Write the matrix M that models this communication network below. Label rows and columns alphabetically.

c) Find M^2. Explain what the element in the fifth row, second column means.

d) Find the matrix that represents the number of ways messages can be sent from one ship to another using at most one relay.

a)



b)

$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

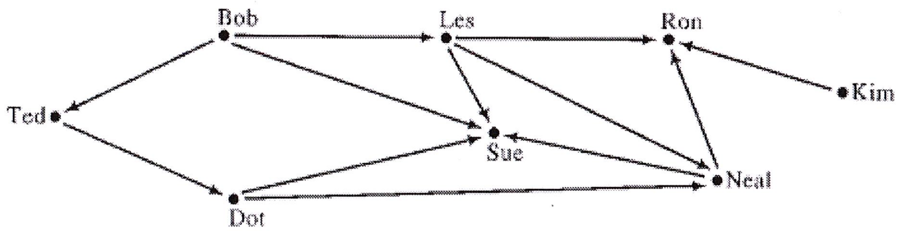
c)

$$M^2 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 2 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 1 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & \boxed{0} & 1 & 0 & 1 \end{bmatrix} \end{array}$$

M^2_{52} means there are zero paths of length 2 from E to B.

d) M^2 shown in part C

6. The diagram below models a rumor network.



If a person has only outgoing arrows, then that person is a transmitter of the rumor. A person having only incoming arrows is a receiver of the rumor. A person having both outgoing and incoming arrows is a relay point for the spreading of a rumor.

a) Identify the transmitters, receivers, and relays for this network.
b) How can you identify a transmitter by looking at the corresponding rumor matrix? How can you identify a receiver?

a) T: Kim, Bob

R: Sue, Ron

Relay: Ted, Dot, Neal, Les

b) The transmitters would have numbers in their row, but not in their column.

Receivers would have numbers in their column, but not their row.

Section 2.5:

1. In a chemical experiment, molecules of a liquid are changing phase in a flask. It is known that from one minute to the next, 80% of the molecules remain liquid, while 20% become gaseous. At the same time, 40% of the gaseous molecules become liquid. The rest remain gaseous.
   a. Give T, the transition matrix.

$$T = \begin{array}{c} \\ L \\ G \end{array}\begin{array}{c} \begin{array}{cc} L & G \end{array} \\ \begin{bmatrix} .80 & .20 \\ .40 & .60 \end{bmatrix} \end{array}$$

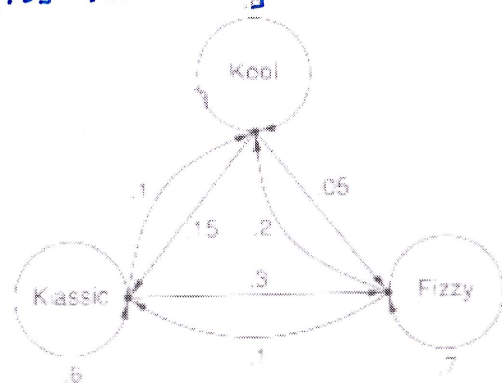
   b. After sufficient time, the substance will reach equilibrium so that the percent that is liquid and the percent this is gaseous become constant. Estimate these percentages by calculating T^8.

$$T^8 : \begin{array}{c} \\ L \\ G \end{array}\begin{array}{c} \begin{array}{cc} L & G \end{array} \\ \begin{bmatrix} .667 & .333 \\ .666 & .334 \end{bmatrix} \end{array}$$

Liquid ≈ 67%

Gas ≈ 33%

2. At a particular time, it was found that 25% of adults drank Kool Cola, 20% drank Klassic Cola, and 55% preferred Fizzy Cola. The Kool Cola Company's surveys showed that people switched brands each month according to the directed graph below.
   a. Based on this data, what percent of adults drank each brand one month after the initial data were gathered? $M_0 \cdot T^1 : \begin{array}{ccc} KC & KLC & FC \\ \end{array} [.33 \quad .2125 \quad .4575]$



Kool: 33% Klassic: 21.25%

Fizzy: 45.75%

$$M_0 = \begin{array}{ccc} KC & KlC & FC \\ \end{array} [.25 \quad .20 \quad .55]$$

$$T = \begin{array}{c} \\ KC \\ KLC \\ FC \end{array}\begin{array}{c} \begin{array}{ccc} KC & KLC & FC \end{array} \\ \begin{bmatrix} .8 & .15 & .05 \\ .1 & .6 & .3 \\ .2 & .1 & .7 \end{bmatrix} \end{array}$$

   b. What percent would eventually drink each brand after a long period of time?

Raise T to a high power!

Kool: 43.9% Klassic: 24.4% Fizzy: 31.7%

$$M_0 T^{20} : \begin{array}{ccc} KC & KLC & FC \\ \end{array} [.439 \quad .244 \quad .317]$$

Section 2.6:

1. Use the table and the process introduced in this lesson to compute the following:

   Initial Female Rat Population

Age (months)	0 – 3	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18
Number	15	9	13	5	0	0

After 3 months (1 cycle):

Age (months)	0 – 3	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18
Number	16.6	9	8.1	11.7	4	0

a. Calculate the number of newborn rats (aged 0 – 3) after 6 months (2 cycles).

$16.6(0)+9(.3)+8.1(.8)+11.7(.7)+4(.4)+0(0) = 18.97$ rats

b. Calculate the number of rats that survive in each age group after 6 months and move up to the next age group.

After 6 months (2 cycles):

Age (months)	0 – 3	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18
Number	18.97	9.96	8.1	7.29	9.36	2.4

c. Use the results of parts a and b to show the distribution of the rat population after 6 months. Approximately how many rats will there be after 6 months?

Sum = 56.08 rats

d. Use your population distribution from part c to calculate the number of rats and the approximate number in each age group after 9 months (3 cycles). Continue this process to find the number of rats after 12 months (4 cycles).

After 9 months (3 cycles):

Age (months)	0 – 3	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18
Number	18.315	11.382	8.964	7.29	5.832	5.616

Total Rats: 57.399 rats

Calculate the number of newborn rats after 12 months (4 cycles) 56.6532

After 12 months (4 cycles):

Age (months)	0 – 3	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18
Number	18.0216	10.989	10.2438	8.0676	5.832	3.4992

Total Rats:

e. Compare the original number of rats with the number of rats after 3, 6, 9, and 12 months. What do you observe?

Went up, but slows down / decreases at the end.

f. What do you think might happen to this population if you extended the calculations to 15, 18, 21, ... months?

The population will eventually level off.

For exercises 2 and 3, use the following birth and survival rates for a certain species of deer:

Age (years)	Birth Rate	Survival Rate
0 – 2	0	0.6
2 – 4	0.8	0.8
4 – 6	1.7	0.9
6 – 8	1.7	0.9
8 – 10	0.8	0.7
10 – 12	0.4	0

2. Use the matrix above:

a. The following table shows a distribution of an initial population of 148 deer.

Age (years)	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12
Number	50	30	24	24	12	8

Find the number of newborn female deer after 2 years (1 cycle).

$50(0) + 30(.8) + 24(1.7) + 24(1.7) + 12(.8) + 8(.4) = 118.4$ deer

b. Calculate the number of deer that survive in each age group after 2 years and move up to the next age group.

Age (years)	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12
Number	118.4	30	24	21.6	21.6	8.4
		50(.6)	30(.8)	24(.9)	24(.9)	12(.7)

c. Arrange the initial population distribution in a row matrix and the birth rates in a column matrix. Multiply the row matrix times the column matrix. Interpret this result.

$$A = \begin{bmatrix} 50 & 30 & 24 & 24 & 12 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ .8 \\ 1.7 \\ 1.7 \\ .8 \\ .4 \end{bmatrix} = \begin{bmatrix} 118.4 \end{bmatrix}$$

newborn deer after one cycle.

3. Explore the possibility of multiplying the initial population distribution in a row matrix times some column matrix to find the number of deer after 2 years that move from:

a. The 0 – 2 group to the 2 – 4 group (HINT: the column matrix that you use will need to contain several zeros in order to produce the desired product.)

$$[A] \cdot \begin{bmatrix} .6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 30$$

b. The 2 – 4 to the 4 – 6 group.

$$[A] \cdot \begin{bmatrix} 0 \\ .8 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 24$$

c. The 4 – 6 to the 6 – 8 group.

$$[A] \cdot \begin{bmatrix} 0 \\ 0 \\ .9 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 21.6$$

d. The 6 – 8 to the 8 – 10 group.

$$[A] \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ .9 \\ 0 \\ 0 \end{bmatrix} = 21.6$$

e. The 8 – 10 to the 10 – 12 group.

$$[A] \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ .7 \\ 0 \end{bmatrix} = 8.4$$

4. Assume an initial deer population of [25 0 0 0 0 0]. Use the birth and survival rate information for the deer population in Exercises 2 and 3 to find the population total and distribution after each of the following time spans.

a. 2 years (1 cycle) $A \cdot B \cdot C = 15$

b. 4 years (2 cycles) $A \cdot B^2 \cdot C = 24$

c. 6 years (3 cycles) $A \cdot B^3 \cdot C = 38.4$

d. 8 years (4 cycles) $A \cdot B^4 \cdot C = 51.84$

$$[A] = \begin{bmatrix} 25 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad [B] = \begin{bmatrix} 0 & .6 & 0 & 0 & 0 & 0 \\ .8 & 0 & .8 & 0 & 0 & 0 \\ 1.7 & 0 & 0 & .9 & 0 & 0 \\ 1.7 & 0 & 0 & 0 & .9 & 0 \\ .8 & 0 & 0 & 0 & 0 & .7 \\ .4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad [C] = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Section 2.7 (You must have a graphing calculator for this assignment):

1. Use the original population distribution and Leslie matrix found in our notes to find the following:
   a. The population distribution after 15 months (5 cycles).

   $P_0 L^5 = [19.47186 \quad 10.81296 \quad 9.8901 \quad 9.21942 \quad 6.45408 \quad 3.4992]$

   b. The total population after 15 months (HINT: Multiply matrix A * matrix B^5 times a column matrix (C) consisting of six 1's.)

   59.34762 rats

   c. The population distribution and the total population after 21 months.

   63.405 rats

2. Suppose the rats start dying off from overcrowding when the total female population for a colony reaches 250. Find how long it will take for this to happen when the initial population is:
   a. $A = [18 \quad 9 \quad 7 \quad 0 \quad 0 \quad 0]$ (Again... Use A * B^x * C on your calculators!)

   Between cycles 60 and 61.

   b. $A = [35 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$

   Between cycles 68 and 69.

   c. $A = [5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5]$

   Between cycles 75 and 76.

3. Again, consider the deer species from Lesson 3. The birth and survival rates follow:

Age (years)	Birth Rate	Survival Rate
0 – 2	0	0.6
2 – 4	0.8	0.8
4 – 6	1.7	0.9
6 – 8	1.7	0.9
8 – 10	0.8	0.7
10 – 12	0.4	0

a. Construct a Leslie Matrix for this animal.

$$\begin{bmatrix} 50 & 30 & 24 & 24 & 12 & 8 \end{bmatrix} \begin{bmatrix} 0 & .6 & 0 & 0 & 0 & 0 \\ .8 & 0 & .8 & 0 & 0 & 0 \\ 1.7 & 0 & 0 & .9 & 0 & 0 \\ 1.7 & 0 & 0 & 0 & .9 & 0 \\ .8 & 0 & 0 & 0 & 0 & .7 \\ .4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b. Given that $P_0 = \begin{bmatrix} 50 & 30 & 24 & 24 & 12 & 8 \end{bmatrix}$, find the long term growth rate.

NOTE: Long term growth rate = $\dfrac{(New\ Population) - (Old\ Population)}{Old\ Population}$

Sample: $\dfrac{P_0L^2C - P_0L^1C}{P_0L^1C} = \dfrac{273.36 - 224}{224} = 22.04\%$

c. Suppose that the natural range for this animal can sustain a herd that contains a maximum of 1,250 females. How long before this herd size is reached?

Between cycles 7 + 8