

SHOW ALL WORK ON A SEPARATE SHEET OF PAPER!

Section 2.1:

1. Using your knowledge of matrices, answer the following:
 - a. How many elements does a 2×8 matrix have? _____
 - b. How many elements does a 1×7 matrix have? _____
 - c. How many elements does an $m \times n$ matrix have? _____

2. A trendy garment shop receives orders from three clothing companies. The first shop orders 25 jackets, 75 shirts, and 75 pairs of pants. The second shop orders 30 jackets, 50 shirts, and 50 pairs of pants. The third shop orders 20 jackets, 40 shirts, and 35 pairs of pants. Display this information in a matrix. Let the rows represent the shops and the columns represent the type of garment ordered. Label the rows and columns of your matrix accordingly.

3. Given the following matrix, answer the questions below:

$$A = \begin{bmatrix} 21 & 14 & 31 \\ 17 & 12 & 5 \\ 8 & 72 & 32 \end{bmatrix}$$

- a. What is the value of A_{21} , A_{12} , and A_{32} ? _____
 - b. If the rows represented shirts, shoes, and pants and the columns represented TJMaxx, JCPenny, and Kohl's, what is the interpretation for entry A_{21} ?
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4. If $A = \begin{bmatrix} 10 & -5 & 13 \\ -7 & 14 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 2 & -8 \\ -4 & 5 & -3 \end{bmatrix}$, find $A + B$ and $B + A$. Is matrix addition **commutative**? (Does $A + B = B + A$?)

5. If $A = \begin{bmatrix} 10 & -5 & 13 \\ -7 & 14 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 2 & -8 \\ -4 & 5 & -3 \end{bmatrix}$, find $A - B$ and $B - A$. Is matrix subtraction **commutative?** (Does $A - B = B - A$?)

6. For breakfast, Bobbi had cereal, a banana, a cup of milk, and a slice of toast. She recorded the following information in her food journal. Cereal: 165 calories, 3 g fat, 33 g carbs, and no cholesterol. Banana: 120 calories, no fat, 26 g carbs, and no cholesterol. Milk: 120 calories, 5 g fat, 11 g carbs, and 15 mg cholesterol. Toast: 125 calories, 6 g fat, 14 g carbs, and 18 mg cholesterol.

- a. Write the information in a matrix N whose rows represent the foods. Label the rows and columns of your matrix.

b. State the values of N_{23} , N_{32} , and N_{41} . _____

c. Write an interpretation of N_{23} . _____

7. Find the value of each of the following expressions or explain why it is not possible.

a. $\begin{bmatrix} 2 & 4 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 7 & -8 \\ -1 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 5 & 7 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 5 & 2 \\ 7 & 4 \end{bmatrix}$

c. $[1 \ 5 \ 10] - \begin{bmatrix} 1 \\ 5 \\ 10 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 3 & 0 \\ 5 & 0 \\ 0 & -4 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ -3 & 0 \\ 2 & -4 \\ 0 & -2 \end{bmatrix}$

e. $\begin{bmatrix} -2y \\ x \end{bmatrix} = \begin{bmatrix} 4 - 5x \\ y + 5 \end{bmatrix}$

f. $\begin{bmatrix} 4x - 1 \\ 9y + 5 \end{bmatrix} = \begin{bmatrix} 3x \\ y - 3 \end{bmatrix}$

8. The matrix M below shows the mileage between 5 major US cities.

	<i>Atlanta</i>	<i>Boston</i>	<i>Chicago</i>	<i>LA</i>	<i>St.Louis</i>
<i>Atlanta</i>	0	1075	716	2211	555
<i>Boston</i>	1075	0	1015	3026	1187
<i>Chicago</i>	716	1015	0	2034	297
<i>LA</i>	2211	3026	2034	0	1842
<i>St.Louis</i>	555	1187	297	1842	0

- $M =$
- Entries that are located in row i , column j , where $i = j$, are said to be located on the **main diagonal** of the matrix. Examine the entries on the main diagonal of M . What do you notice?
 - A square matrix R with order $n \times n$ is **symmetric** if $R_{ij} = R_{ji}$. Is matrix M symmetric? Explain.
 - Give an example of a 3×3 matrix that is symmetric.

 - Could a matrix that is not square ever be symmetric? Why?

Section 2.2:

1. The regents at a state university recently announced a 7% raise of tuition rates per semester hour. The current rates per semester hour are shown in the following table:

	Undergraduate	Graduate
Resident	\$75.00	\$99.25
Non-resident	\$204.00	\$245.25

- a. Write and label a matrix that represents the table above.
- b. Find a new matrix that represents the tuition rates per semester hour after the 7% raise goes into effect. Label your matrix.
- c. Find a matrix that represents the dollar increase for each of the categories. Label your matrix.
2. For each of the following state whether the matrix product QP is defined. If so, give the order (dimensions) of the product.
- a. Order of Q: 1×4 . Order of P: 4×2
 - b. Order of Q: 1×3 . Order of P: 2×3
 - c. Order of Q: 1×5 . Order of P: 5×4
 - d. Order of Q: 1×2 . Order of P: 4×2
 - e. Order of Q: $1 \times m$. Order of P: $m \times 1$

3. Use the following matrices to compute the given expressions.

$$E = \begin{bmatrix} 3 & 8 & -1 \\ 2 & 0 & 4 \end{bmatrix}, F = \begin{bmatrix} 2 & 4 \end{bmatrix}, \text{ and } G = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

- a. $3E$ d. FE
- b. FG e. EF
- c. $-2G$

4. A hobby shop has three different locations: North, South, and East. The store's sales for July are shown in the following table.

	North	South	East
Model trains	10	8	12
Model cars	6	5	4
Model planes	3	2	2
Model trucks	4	3	2

Suppose that model trains sell for \$40 each, cars for \$35, planes for \$80, and trucks for \$45. Use matrix multiplication to find the shop's total sales at each location. Label your matrices.

5. Using the definitions of matrix multiplication, answer the following:
- What must be true about the dimensions of matrices A and B if the product $C = AB$ is defined?
 - If the products AB and BA are both defined, what must be true about the dimensions of matrices A and B ? Why?
 - Find two nonsquare matrices A and B , where AB and BA are both defined. Compute AB and BA . Are they equal? Why?

6. Given the matrices A , B , and C :

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \\ 2 & 2 \\ -1 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

- Do you think that $A(BC) = (AB)C$?
- Test your conjecture by computing the products $A(BC)$ and $(AB)C$.
- The computations in part b show one case in which **matrix multiplication is associative**. Do you think this property holds for all matrices A , B , and C for which the product $A(BC)$ is defined? Why or why not?

Section 2.3:

1. Recall the definition of the **transpose** of matrix A .

a. Describe the transpose of a row matrix.

b. Describe the transpose of a column matrix.

c. Write the transpose of matrix E (E^T) if $E = \begin{bmatrix} 3 & 8 & -1 \\ 2 & 0 & 4 \end{bmatrix}$.

2. Using matrix A and B below:

a. Verify that the matrices A and B are inverses of each other by computing AB and BA .

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

b. Not all square matrices will have an inverse. Use algebra to show that matrix C does not have an inverse.

$$C = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$$

c. If you multiply the diagonals of matrix C and subtract their products, what do you get?

3. Determine whether each pair of matrices are inverses or not.

a. $X = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ c. $X = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}, Y = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

b. $M = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}, N = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$

4. Find the inverse of each matrix, if it exists.

a. $\begin{bmatrix} 0 & 2 \\ 4 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 1 & -1 \\ 3 & 3 \end{bmatrix}$

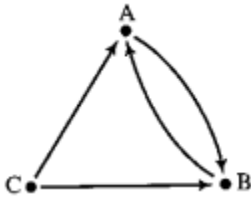
b. $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

d. $\begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$

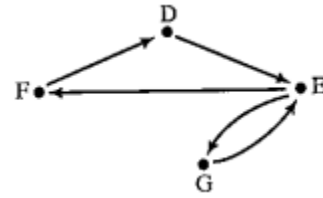
Section 2.4:

Write the communication matrix for each communication network given below:

1.



2.



Draw a communication network that is described by the given matrices below:

3.

$$\begin{array}{c}
 \text{A} \\
 \text{B} \\
 \text{C} \\
 \text{D}
 \end{array}
 \begin{bmatrix}
 \text{A} & \text{B} & \text{C} & \text{D} \\
 0 & 1 & 0 & 0 \\
 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 \\
 1 & 1 & 1 & 0
 \end{bmatrix}$$

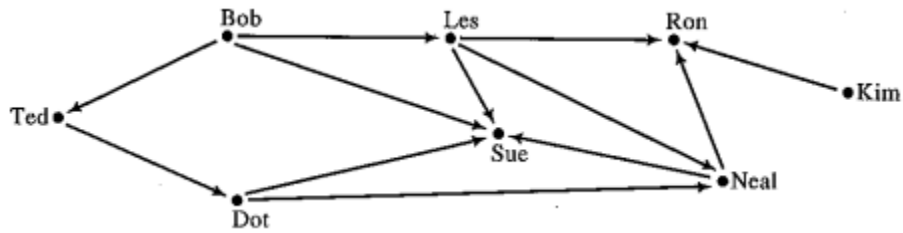
4.

$$\begin{array}{c}
 \text{R} \\
 \text{S} \\
 \text{T} \\
 \text{U} \\
 \text{V}
 \end{array}
 \begin{bmatrix}
 \text{R} & \text{S} & \text{T} & \text{U} & \text{V} \\
 0 & 0 & 1 & 0 & 1 \\
 1 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & 1 & 0 & 1 \\
 0 & 0 & 0 & 1 & 0
 \end{bmatrix}$$

5. Answer the questions about a shipping scenario below:

- Ship A sends messages to Ship B. Ship B sends and receives messages from Ships C and E. Ship D sends and receives messages from Ships A and C. Draw this network below.
- Write the matrix M that models this communication network below. Label rows and columns alphabetically.
- Find M^2 . Explain what the element in the fifth row, second column means.
- Find the matrix that represents the number of ways messages can be sent from one ship to another using at most one relay.

6. The diagram below models a rumor network.



If a person has only outgoing arrows, then that person is a transmitter of the rumor. A person having only incoming arrows is a receiver of the rumor. A person having both outgoing and incoming arrows is a relay point for the spreading of a rumor.

- Identify the transmitters, receivers, and relays for this network.
- How can you identify a transmitter by looking at the corresponding rumor matrix? How can you identify a receiver?

Section 2.6:

1. Use the table and the process introduced in this lesson to compute the following:

Initial Female Rat Population

Age (months)	0 – 3	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18
Number	15	9	13	5	0	0

After 3 months (1 cycle):

Age (months)	0 – 3	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18
Number	16.6	9	8.1	11.7	4	0

- Calculate the number of newborn rats (aged 0 – 3) after 6 months (2 cycles).
- Calculate the number of rats that survive in each age group after 6 months and move up to the next age group.

After 6 months (2 cycles):

Age (months)	0 – 3	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18
Number						

- Use the results of parts a and b to show the distribution of the rat population after 6 months. Approximately how many rats will there be after 6 months?
- Use your population distribution from part c to calculate the number of rats and the approximate number in each age group after 9 months (3 cycles). Continue this process to find the number of rats after 12 months (4 cycles).

After 9 months (3 cycles):

Age (months)	0 – 3	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18
Number						

Total Rats:

Calculate the number of newborn rats after 12 months (4 cycles) _____

After 12 months (4 cycles):

Age (months)	0 – 3	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18
Number						

Total Rats:

- e. Compare the original number of rats with the number of rats after 3, 6, 9, and 12 months. What do you observe?
- f. What do you think might happen to this population if you extended the calculations to 15, 18, 21, ... months?

For exercises 2 and 3, use the following birth and survival rates for a certain species of deer:

Age (years)	Birth Rate	Survival Rate
0 – 2	0	0.6
2 – 4	0.8	0.8
4 – 6	1.7	0.9
6 – 8	1.7	0.9
8 – 10	0.8	0.7
10 – 12	0.4	0

2. Use the matrix above:
 - a. The following table shows a distribution of an initial population of 148 deer.

Age (years)	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12
Number	50	30	24	24	12	8

Find the number of newborn female deer after 2 years (1 cycle).

- b. Calculate the number of deer that survive in each age group after 2 years and move up to the next age group.

Age (years)	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12
Number						

- c. Arrange the initial population distribution in a row matrix and the birth rates in a column matrix. Multiply the row matrix times the column matrix. Interpret this result.

3. Explore the possibility of multiplying the initial population distribution in a row matrix times some column matrix to find the number of deer after 2 years that move from:
 - a. The 0 – 2 group to the 2 – 4 group (HINT: the column matrix that you use will need to contain several zeros in order to produce the desired product.)

 - b. The 2 – 4 to the 4 – 6 group.

 - c. The 4 – 6 to the 6 – 8 group.

 - d. The 6 – 8 to the 8 – 10 group.

 - e. The 8 – 10 to the 10 – 12 group.

4. Assume an initial deer population of $[25 \ 0 \ 0 \ 0 \ 0 \ 0]$. Use the birth and survival rate information for the deer population in Exercises 2 and 3 to find the population total and distribution after each of the following time spans.
 - a. 2 years (1 cycle)

 - b. 4 years (2 cycles)

 - c. 6 years (3 cycles)

 - d. 8 years (4 cycles)

3. Again, consider the deer species from Lesson 3. The birth and survival rates follow:

Age (years)	Birth Rate	Survival Rate
0 – 2	0	0.6
2 – 4	0.8	0.8
4 – 6	1.7	0.9
6 – 8	1.7	0.9
8 – 10	0.8	0.7
10 – 12	0.4	0

a. Construct a Leslie Matrix for this animal.

b. Given that $P_0 = [50 \ 30 \ 24 \ 24 \ 12 \ 8]$, find the long term growth rate.

NOTE: Long term growth rate =
$$\frac{(New \ Population) - (Old \ Population)}{Old \ Population}$$

c. Suppose that the natural range for this animal can sustain a herd that contains a maximum of 1,250 females. How long before this herd size is reached?