**Chapter 1 – Election Theory Notes Packet** **Name__________________________**

**<u>Section 1.1 - Decision Making:</u>**

In the last 24 hours, what are some decisions you have had to make? List as many as you can.

Decisions that people make can have important consequences (positive and negative). For example, Nielsen Media Research polls individuals to learn what tv programs they decide to watch. These decisions determine whether or not a show will survive another season. Organizations like these have to combine the preferences of all individuals in their survey into a single result, and they have a responsibility to the viewing audiences to do so in a way that is fair to all tv programs.

<u>Think about this…</u>

1. How are the wishes of many individuals combined to yield a single result?
2. Do the methods for doing so always treat each choice fairly?
3. If not, is it possible to improve on these methods?

**<u>ACTIVITY</u>**

You are going to fill out a form, ranking the following choices of pop from your first choice to your fifth choice: Pepsi, Mountain Dew, Root Beer, Dr. Pepper, and Sprite

Once we have all of the data from our survey, you'll be trying to summarize all the data into a final class ranking for our class' first through fifth choices.

***Your task:***

1. ***Individually*** devise a method of combining the rankings of all the individuals in your class into a single ranking for the entire class. Your method should produce a first, second, third, fourth, and fifth place ranking for each pop listed. Record your thoughts below.

2. With your group, each person needs to share the method they used to arrive at their ranking for the class. Try to come to an agreement at your table of the 'best method'.

3. Report out to the class. Present your final ranking to the class and describe the method used to obtain it. Clear communication of the method used to obtain a result is important in mathematics, so everyone should strive for clarity when making the presentation.

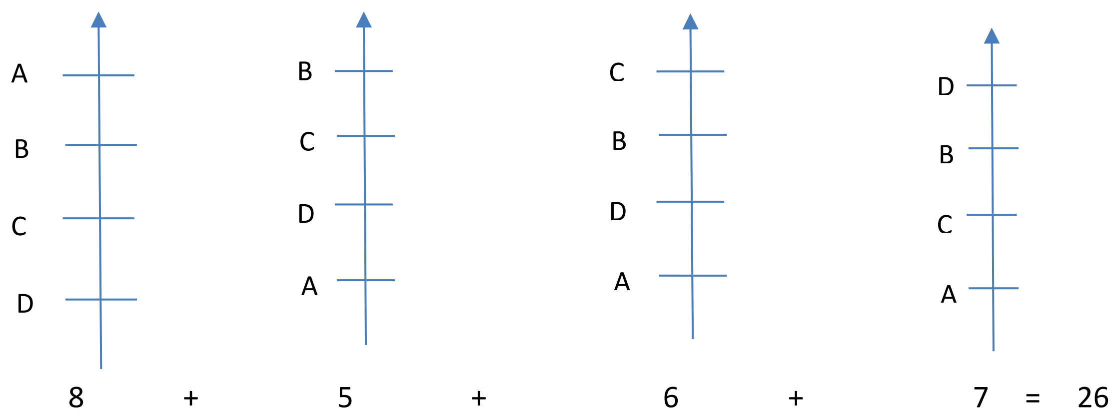
In order to be able to prepare answer keys ahead of time, you'll use the following group rankings when referenced throughout the Chapter 1 homework:

| **Group 1 (5 people)** | **Group 2 (6 people)** | **Group 3 (6 people)** | **Group 4 (6 people)** |
|---|---|---|---|
| Pepsi | Sprite | Dr. Pepper | Pepsi |
| Mountain Dew | Pepsi | Pepsi | Sprite |
| Sprite | Mountain Dew | Sprite | Dr. Pepper |
| Dr. Pepper | Dr. Pepper | Mountain Dew | Mountain Dew |
| Root Beer | Root Beer | Root Beer | Root Beer |

**Section 1.2 - Group-Ranking Methods and Algorithms:**

Even among professionals, there is rarely a consensus on group ranking without controversy. This lesson will examine several common methods of determining a group ranking from a set of voter preferences. As we examine these methods, consider whether any of them are similar to the ones devised by members of your class in section 1.

Consider the preferences from our last lesson:




Many voting situations, such as elections in which there is only one office to fill, require the selection of a single winner. Although most such elections in the US do not use a preferential ballot, they could.

Example: In the set of preferences shown, choice A is ranked first on eight schedules, more often than any other choice. If A wins on this basis, A is called the **plurality winner**. The plurality winner is based on first place rankings only.

What percentage of the votes did A win?________________ If A is first on over half of the

schedules, A is considered a ___________________________.

If you determine ranking by assigning points to the first, second, third, fourth, etc. choice of each individual's preference and obtain a point total, you are using a **Borda count**. (If there are 4 options, first place would get 4 points, $2^{nd}$ would get 3, etc.)

Using a Borda count for the preferences listed on page 4:

A:

B:

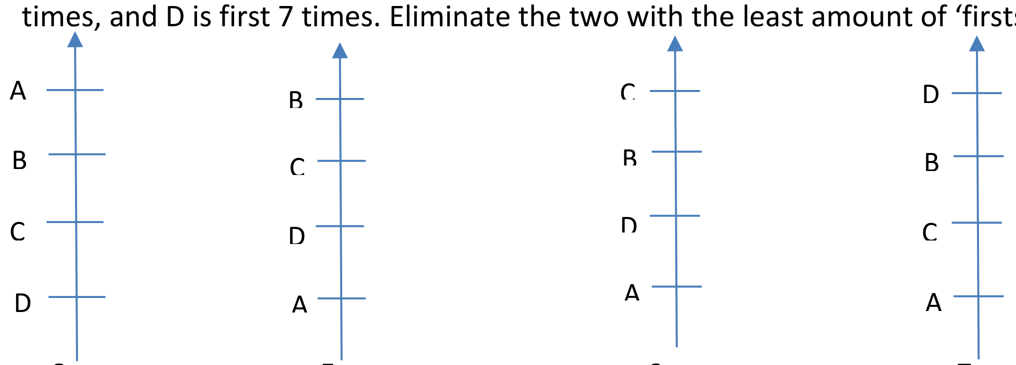
C:

D:

When using a Borda system, who "wins" the election in our example?

**<u>Runoff Methods</u>**

When no 'majority' winner is found, sometimes a runoff election will need to be held with the top two candidates. To conduct a runoff election, you need to determine the number of 'firsts' for each choice. In our example above, A is first 8 times, B is first 5 times, C is first 6 times, and D is first 7 times. Eliminate the two with the least amount of 'firsts': B and C.




8 + 5 + 6 + 7 = 26

The result?

## Sequential Runoff Method

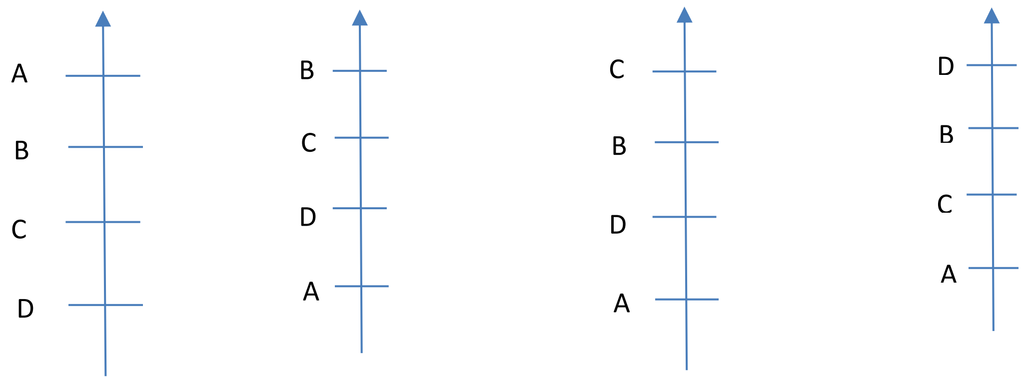
Similar to the runoff method, the sequential runoff method only eliminates ONE choice at a time. Since B is ranked first the fewest number of times, it would be the choice to be eliminated. How does that affect the vote?

Now take the 2nd smallest total, and do the elimination method again. Now who is the winner?

**Section 1.3: Condorcet's Method**

As we have seen, different methods of determining a group ranking can give different results. Mathematician Marquis de Condorcet proposed that a choice that could obtain a majority over every other choice should be ranked first for that group.

Consider the set of preference schedules used in our last lesson.




8 + 5 + 6 + 7 = 26

To examine these data for a **Condorcet winner**, we need to compare each choice with every other choice. Let's do some comparisons….

A vs. B

A vs. C

A vs. D

B vs. C

B vs. D

C vs. D

Who is the **Condorcet winner** for this set of data, if there is one?

Using the schedules below, who would be the **Condorcet winner**, if there is one?

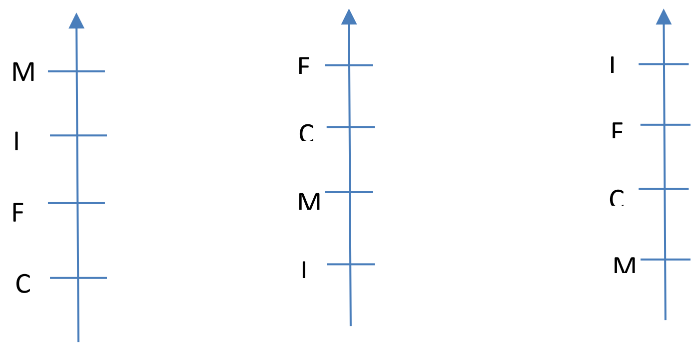
| # of Voters | Preference Order |
|---|---|
| 14 | B > C > D > A |
| 10 | C > D > B > A |
| 9 | A > D > B > C |
| 6 | A > C > B > D |
| 4 | D > C > B > A |

In Geometry, we talked about the transitive property (If a > b and b > c, then a > c). What you should notice here is that group-ranking methods may violate the transitive property. Because this is contrary to what you would believe SHOULD happen, we call it a **paradox** (specifically, this one is called the **Condorcet paradox**).

## Section 1.4: Arrow's Conditions and Approval Voting

A lot of work has been done to attempt to improve the group-ranking process. First, let's consider an example involving pairwise voting.

*Ten representatives of the language clubs at Central High School are meeting to select a location for the clubs' annual joint dinner. The committee must choose among a Chinese (C), French (F), Italian (I), or Mexican (M) restaurant.*




4 + 3 + 3 = ___________

Rachel says that because the last two dinners were at Mexican and Chinese restaurants, this year's dinner should be at either an Italian or French restaurant. The group votes 7:3 in favor of Italian.

Martin, who doesn't like Italian food, says that the community's newest Mexican restaurant has an outstanding reputation. He proposes that the group choose between Italian and Mexican. The other members agree and vote 7:3 to hold the dinner at the Mexican restaurant.

Sarah, whose parents own a Chinese restaurant, says that she can obtain a substantial discount for the event. The group votes between the Mexican and Chinese restaurants and selects the Chinese by a 6:4 margin.

Based on this voting scenario, Chinese wins out in the end. Look carefully at the group members' preferences. Which type of restaurant was ultimately preferential to most students?

Paradoxes like this led Kenneth Arrow (a US Economist) to formulate 5 conditions necessary for a fair group ranking method. These methods are called **Arrow's Conditions**.

**Arrow's Conditions**

1. **Nondictatorship -** the preferences of a ________________________________ should not become the group ranking without considering ______________________________.

2. **Individual Sovereignty -** each ____________________ should be allowed to order the choices ______________________ and to indicate ___________.

3. **Unanimity -** if _____________________________ prefers one choice to another, then the group ranking should do the same.

4. **Freedom from Irrelevant Alternatives –** if a choice is _________________, the order in which the others are ranked ________________________. (A removed choice = ____________________________).

5. **Uniqueness of the Group Ranking –** the method of producing the group ranking should give the _________________________ whenever it is applied to a given set of preferences.

## Section 1.5: Weighted Voting and Voting Power

In some voting situations, some people may have more votes than other voters (think Electoral College).

Consider this example: A small high school ($10^{th}$ – $12^{th}$ grade) has 110 students. Because of recent growth in the size of the community, the sophomore class is quite large: 50 members. The junior and senior classes both have 30 members. The school's student council is composed of a single representative from each class. Each of the three members is given a number of votes proportionate to the size of the class represented. Accordingly, the sophomore representative has five votes, and the junior and senior representatives each have 3 votes. The passage of any issue that is before the council requires a simple majority (6 votes).

This scenario is an example of **weighted voting**. John Banzhaf, a law professor, has initiated many legal actions against weighted voting procedures used in local government. To understand his objection, consider the number of ways that voting on an issue could occur in the student council example. List the possibilities below:

Each of these collections of voters is called a **coalition**. Those with enough votes to pass an issue are known as **winning coalitions**. List the winning coalitions in this example (those with 6 or more votes):

Note: When everyone votes favorably, the total votes is 11. If one member decides to vote differently, the outcome remains the same. No single member is essential to the coalition. Banzhaf argued that the only time a voter has power is when the voter belongs to a coalition that needs the voter in order to pass an issue. The coalitions for which at least one member is essential are:

How often is the sophomore representative essential?

The junior representative? The senior representative?

*<u>The paradox:</u>* Although the votes have been distributed to give greater power to the sophomores, the outcome is that all members have the same power! Since distributing the votes in a way that reflects the population distribution does not always result in a fair distribution of power, mathematical procedures can be used to develop ways to measure actual power in weighted voting situations.

A measure of the power of a member of a voting body is called a **power index**. In this lesson, a voter's power index is the number of winning coalitions in which the voter is essential. For example, in the student council situation, the sophomore representative is essential to two winning coalitions, and thereby has a power index of 2, as do the junior and senior representatives.

**<u>A Power Index Algorithm</u>**

1. List all coalitions of voters that are winning coalitions.
2. Select any voter, and record a 0 for that voter's power index.
3. From the list in step 1, select a coalition of which the voter selected in step 2 is a member. Subtract the number of votes the voter has from the coalition's total. If the result is less than the number of votes required to pass an issue, add 1 to the voter's power index.
4. Repeat step 3 until all coalitions of which the voter chosen in step 2 is a member are checked.
5. Repeat steps 2 – 4 until all voters are checked.