**1.1 Assignment:**

1. Do all the group rankings produced in your class have the same pop ranked first? If not, which pop is ranked first most often?

2. Second?

3. Third?

4. Fourth?

5. Fifth?

6. Write a description of the method *you* used to achieve a group ranking. Make it clear enough that another person can use the method.

7. Did your method result in any ties? If it did (or could), how could you modify your method to break any ties?

8. Mathematicians often find it convenient to represent a situation in a compact way. A good representation conveys the essential information about a situation. In election theory, a **preference schedule** is sometimes used to represent the preferences of one or more individuals. The following preference schedule displays four choices, called A, B, C, and D. It indicates that the individual whose preference it represents ranks B first, C second, D third, and A fourth.

B
C
D
A

Since there are often several people who have the same preferences, mathematicians write the number of people or the percentage of people who expressed that preference under the schedule. The preferences in a group of 26 people are represented by the preference schedules listed below.

A		B		C		D		
B		C		B		B		
C		D		D		C		
D		A		A		A		
8	+	5	+	6	+	7	=	26
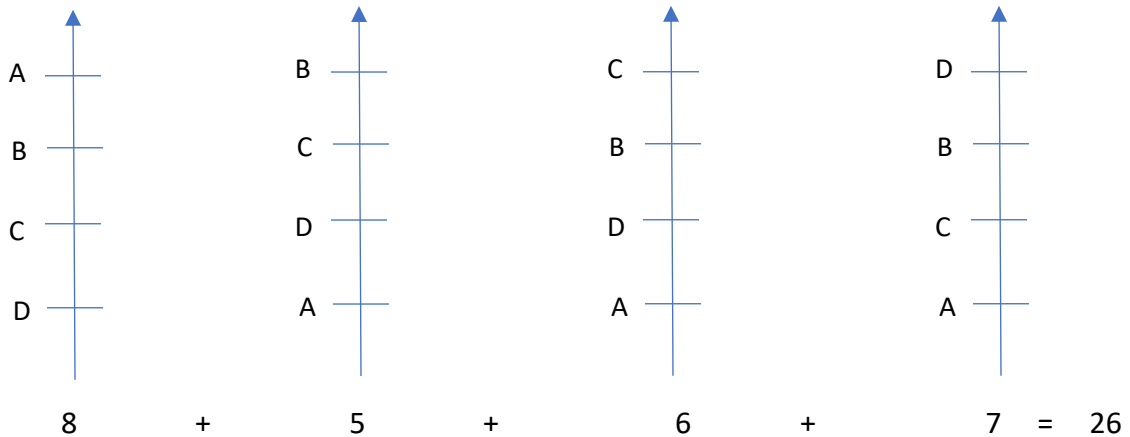
a) Apply the method you used to determine your class's pop rating to this set of preferences. List the first, second, third, fourth, and fifth place rankings that your method produces.

b) Do you think the ranking your method produces is fair? If you worked in a group, do all members of your group think the result is fair? In other words, do the first, second, third, and fourth place rankings seem reasonable, or are there reasons that one or more of the rankings seem unfair? Explain.

c) Would preference schedules be a useful way to represent the individual preferences for pop flavors among the members of your class? Explain.

9. When your class members voted on pop flavors, they ranked them from the first through fifth. A ballot that allows voters to rank the choices is called a **preferential ballot**. In most elections in the US, preferential ballots are not used. Do you think preferential ballots are a good idea? Explain.

1.2 Assignment:

1. Find a plurality winner for the pop voting from the top of page 3 of your notes. Is it also a majority winner? Explain.

2. Find the Borda winner for the pop voting from our class.

3. Find a runoff winner for the pop voting from our class.

4. Find a sequential runoff winner for the pop voting from our class.

5. The International Olympic Committee uses sequential runoff voting to choose Olympic sites. Since the committee is relatively small, separate rounds of voting are used rather than a single round with preferential ballots. There were three cities competing to host the 2010 Winter Games. This table summarizes the voting.

Vancouver	40	56
Pyeongchang	51	53
Salzburg	16	

Write a short summary of the voting, including the order in which the cities were eliminated.

6. For our example from this section on page 4 of your notes, determine the percentage of voters that rank each choice first and last.

    a. Record the results in the table:

 | Choice | First | Last |
    |--------|-------|------|
    | A | | |
    | B | | |
    | C | | |
    | D | | |

    Use this table to answer the questions below.

    b. On the basis of these percentages only, which choice do you think is most objectionable to voters? Least objectionable? Explain your answers.

    c. Which choice do you think most deserves to be ranked first for the group? Explain your reasoning.

    d. Give at least one argument against your choice.

7. Determine the plurality, Borda, runoff, and sequential runoff winners for the following set of preferences.

16	20	12	7
A	B	C	C
D	D	B	D
C	A	D	A
B	C	A	B

16   +   20   +   12   +   7   =   _____
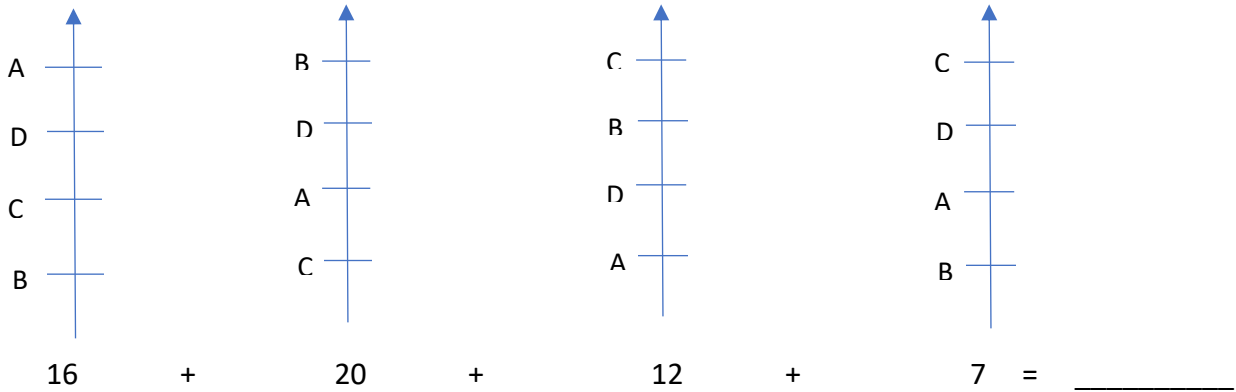
Plurality:

Borda:

Runoff:

Sequential Runoff:

8. Each year the Heisman Trophy recognizes one of the country's outstanding college football players. In 2004, Southern California quarterback Matt Leinart received the award. The results of the voting follow. Each voter selects a player to rank first, another to rank second, and another to rank 3rd.

	First	Second	Third	Points Total
Matt Leinart, USC	267	211	102	1325
Adrian Peterson, Oklahoma	154	180	175	997
Jason White, Oklahoma	171	149	146	957
Alex Smith, Utah	98	112	117	635
Reggie Bush, USC	118	80	83	597
Cedric Benson, Texas	12	41	69	187
Jason Campbell, Auburn	21	24	51	162
JJ Arrington, California	10	33	19	115
Aaron Rodgers, California	8	14	15	67
Braylon Edwards, Michigan	3	13	27	62

a. How many points are awarded for a first place vote? Second place vote? Third place vote?

b. Does the ranking produced by this system differ if a plurality method is used? Explain.

1.3 Assignment:

1. Who is the Condorcet winner of our pop tally from section 1 (page 3 of notes)? Show work here.

2. A panel of sportswriters is selecting the best football team in a league, and the preferences are distributed as follows.

| A	B	C
B	A	B
C	C	A
52	38	10
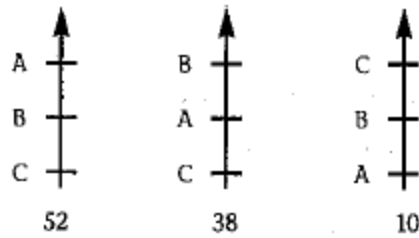
   a. Determine a best team using a 3-2-1 Borda count.

   b. The 38 who rank B first and A second decide to lie in order to improve the chances of their favorite and so rank C second. Determine the winner using a 3-2-1 Borda count.

3. Many people consider plurality methods unfair because they can produce a winner that a majority of voters do not like.

   a. What percentage of voters ranks the plurality winner last in the preferences shown below?

A	C	B	D
B	B	D	E
D	E	E	C
C	D	C	B
E	A	A	A
20	18	15	12
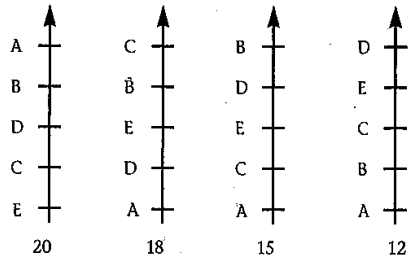
b. Runoffs are sometimes used to avoid the selection of a controversial winner. Is a runoff winner an improvement over a plurality winner in this set of preferences? Explain.

c. Do you consider a sequential runoff winner an improvement over plurality and runoff winners? Explain.

4. Use a 4-3-2-1 Borda count to determine a group ranking for the following set of preferences.

A	B
C	D
B	A
D	C
50	40
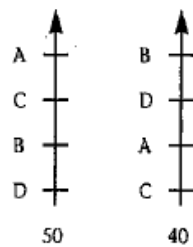
a. These preferences represent the ratings of four college athletic teams, and team C has been disqualified because of a recruiting violation. Write the schedules with team C removed and use a 3-2-1 Borda count to determine a group ranking.

b. Explain why these results are a paradox.

5. US College Hockey Online (USCHO) has several ranking systems. In a system called pairwise ranking, USCHO compares each team to every other team. In each comparison, the team that compares favorably to the other is awarded a point. The team with the most points is ranked first. Consider a simple version of this system in a league with 6 teams, A, B, C, D, E, and F. The following table shows the results of the comparisons. An X in a team's row indicates that it won the comparison with the team at the top of the column.

	A	B	C	D	E	F
A		X	X	X	X	
B			X	X		
C				X	X	
D					X	X
E		X				X
F	X	X	X			

   a. Find two pairwise comparisons in this table that demonstrate the transitive property.

   b. Find two comparisons that demonstrate a violation of the transitive property.

   c. If each team receives a point for each comparison that it wins, find a group ranking for these teams.

   d. Suggest a modification to the point system that is advantageous to team F.

   e. Suppose team D drops out of the league. What effect does this have on the rankings you found in part c?

1.4 Assignment:

1. Mr. Klassen decides to order pop for the class on the basis of the pop vote from section 1, but in doing so, he selects the preference schedule of a single student (the teacher's pet, of course!). Which of Arrow's conditions are being violated by this method of determining a group ranking?

2. Instead of selecting the preference schedule of a favorite student, Mr. Klassen places all the individual preferences in a hat and draws one. If this method were repeated, would the same group ranking result? Which of Arrow's conditions does this method violate?

3. You have seen situations in which insincere voting occurs. Do any of Arrow's conditions state that insincere voting should not be part of a fair group-ranking method? Explain.

4. Suppose that there are only two choices in a list of preferences and that the plurality method is used to decide the group ranking. Which of Arrow's conditions could be violated? Explain.

1.5 Assignment:

1. Consider a situation in which A, B, and C have 3, 2, and 1 votes respectively, and in which 4 votes are required to pass an issue.
    a. List all possible coalitions and all winning coalitions.

    b. Determine a power index for each voter.

    c. If the number of votes required to pass an issue is increased from 4 to 5, determine a power index for each voter.

2. In a situation with three voters, A has 7 votes, B has 3, and C has 3. A simple majority is required to pass an issue.
    a. Determine a power index for each voter.

    b. A *dictator* is a member of a voting body who has all the power. A *dummy* is a member who has no power. Are there any dictators or dummies in this situation?

3. In this lesson's student council example, suppose that the reps of the junior and senior classes always differ on issues and never vote alike. Does this make any practical difference in the power of the three representatives? Explain.

4. One way to determine all winning coalition is a weighted voting situation is to work from a list of all possible coalitions. Use A, B, C, and D to represent the individuals in a group of four voters and list all possible coalitions.

5. Weighted voting is commonly used to decide issues at meetings of corporate stockholders. Each member has one vote for each share of stock held.
   a. A company has four stockholders: A, B, C, and D. They own 26%, 25%, 25%, and 24% of the stock respectively, and more than 50% of the vote is needed to pass an issue. Determine a power index for each stockholder (use your results from #4 as an aid).

   b. Another company has four stockholders. They own 47%, 41%, 7%, and 5% of the stock. Find a power index for each stockholder.

   c. Compare the percentage of stock owned by the smallest shareholder in parts a and b. Do the same for the power index of the smallest stakeholder in each case.

Unit 1: Election Theory Test Review

1. An election is being held to choose the president of the Math Advisory Club. There are 5 candidates: A, B, C, D, and E. Each of the 22 members of the club is asked to submit a ballot indicating his or her first, second, third, fourth, and fifth choices; ties are not allowed on individual ballots. The 22 ballots submitted are summarized in the table below:

First choice	A	B	C	E
Second choice	D	C	D	D
Third choice	C	E	B	C
Fourth choice	E	D	E	B
Fifth choice	B	A	A	A
Number of Voters	**4**	**6**	**7**	**5**

a) Who is the plurality winner?   a) _____

b) Who is the winner by the runoff method?   b) _____

   Who was eliminated?_____

c) Who is the winner by the sequential runoff method?   c) _____

   Who was eliminated, and in what order?

d) Who is the winner by the 5-4-3-2-1 Borda count method?   d) _____
   A:

   B:

   C:

   D:

   E:

e) Suppose that the election is by the approval voting method and that each e) _____
   voter only approves of the top three candidates on his or her preference list.
   Who is the winner in this case?

f) Who is the Condorcet winner, if one exists?   f) _____

g) The club treasurer decides to use a 7-5-4-2-1 Borda count because she   g) _____
   feels that the difference in opinion decreases as voters near the bottom
   of their preference schedules. What are the results of the election using
   this method?

2. An election was held in the Prom Committee Meeting to find a venue for their Spring Prom. The choices are: VanAndel Museum, Civic Center, Frederik Meijer Gardens, and WOHS.

First choice	V	W	C	F
Second choice	F	C	V	V
Third choice	C	V	F	W
Fourth choice	W	F	W	C
Number of Voters	**10**	**7**	**9**	**11**

a) What percentage of first place votes are received by each venue?

VanAndel: _____ Civic: _____ Frederik: _____ WOHS: _____

b) What percentage of last place votes are received by each venue?

VanAndel: _____ Civic: _____ Frederik: _____ WOHS: _____

c) Which location is least objectionable? _____

d) Which location is most objectionable? _____

e) Using plurality voting in a contest with four alternatives, what is the minimum percentage of votes that an alternative must receive in order to win?

3. Consider the weighted voting situation in which voters A, B, C, and D have 10, 7, 5, and 3 votes respectively. 13 votes are needed to pass an issue.

a) Is the coalition formed by A, C, and D a winning coalition? _____

b) Which players are essential in the coalition {A, B, D; 20}? _____

c) Which players are essential in the coalition {A, B, C, D; 25}? _____

d) List all of the winning coalitions below:

e) Find the power index for each voter.

A = _____   B = _____   C = _____   D = _____