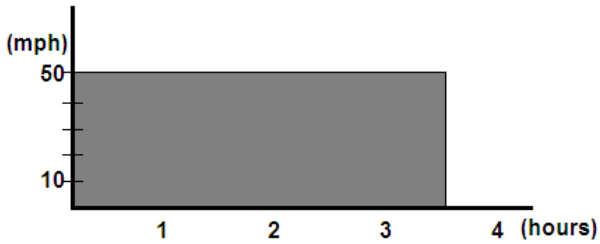
15.9 Area Under the Curve (Riemann)

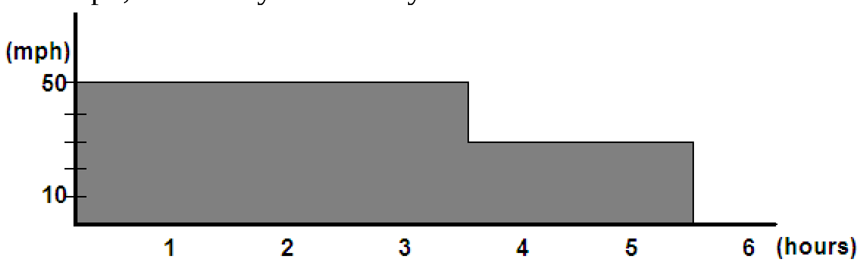
Objectives:

- Find areas under graphs of polynomial functions.

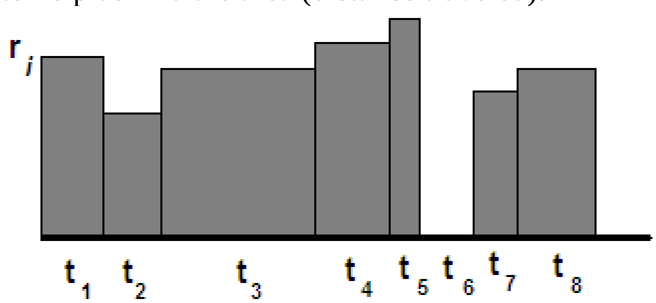
Example1:
If you travel for 3.5 hours at a constant speed of 50 mph, how far did you travel? Remember that the formula $d = rt$ gives the total distance traveled.



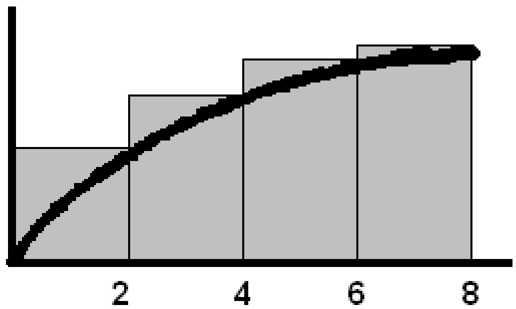
Example 2:
If you travel 3.5 hours at a constant speed of 50 mph and then 2 hours at a constant speed of 30 mph, how many miles did you travel?



Since we know it is nearly impossible to travel at a "constant" speed, our driving situation would most likely look more like the picture below. We can use sigma notation to help us find the area (distance traveled).



Example 3:
A car accelerates from 0 to 88 feet per second with a speed of $g(x) = -.88(x - 10)^2 + 88$ feet per second after x seconds. Estimate the distance that the car travels in 8 seconds by dividing the graph into 4 sub-intervals.



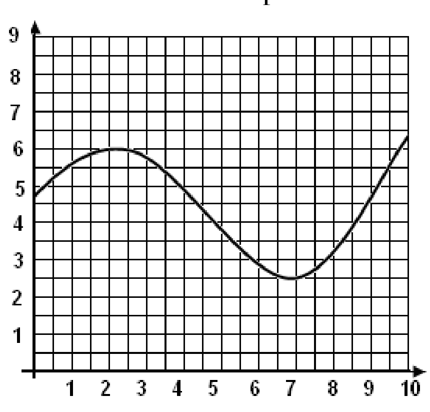
$-.88(x - 10)^2 + 88$

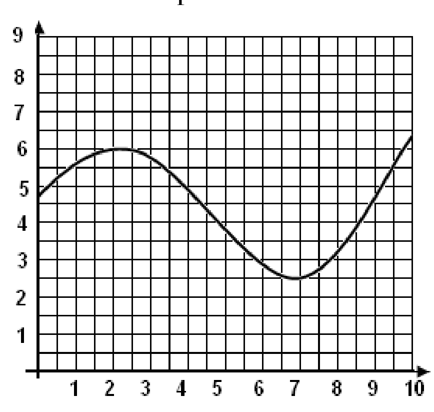
Example 4:
A car accelerates from 0 to 60 miles per hour (88 feet per second) in 10 seconds. If the acceleration is constant, how far will the car travel (in feet) in this time? (Sketch a picture).

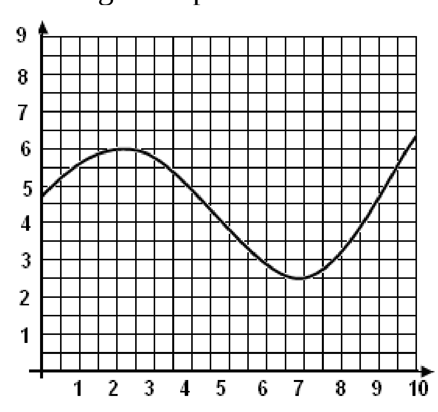
Riemann Sums

Example 5:
A graph of $h(x)$ is given below. Break the interval from 0 to 10 into 5 subintervals of equal length. Evaluate $\sum_{i=1}^{5} h(z_i)$, estimating each $h(z_i)$ to the nearest integer when using:

a. Left endpoint: b. Mid-point: c. Right endpoint:







15.10 *Exact* Area Under the Curve

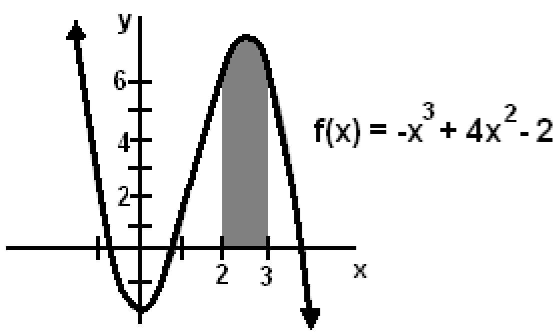
Anatomy of an integral:

$$\int_a^b f(x)dx$$

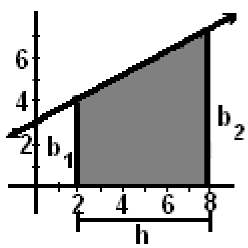
Objectives:

- Find the **exact** area under a curve.
- Find antiderivatives of functions.
- Find indefinite integrals of polynomial functions.

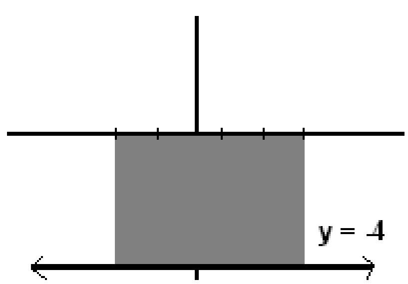
Example 1:
Express the area of the shaded region below with an integral.



Example 2:
Evaluate: $\int_2^8 \left(\frac{1}{2}x+3\right)dx$



Example 3:
Find the exact area of $\int_{-2}^{3}(-4)dx$.



Finding an "anti-derivative":

Evaluating the integral without a graph is needed when we are looking for the area underneath a curve. For example, finding the area under $y = x^2$ would be impossible to do without using approximations. This is where we would use an **antiderivative (indefinite integrals).**

Power Rule: $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

Although it looks complicated, the process is really quite simple… try some ☺

Example 5:

a. $\int 5x^2 dx$

b. $\int (4x^5 + 7x^2 - 4x) dx$

c. $\int 2x^3 - 4x^2 + 7x - 4 dx$

15.11 The Fundamental Theorem of Calculus

Objectives:
- Use the Fundamental Theorem of Calculus to evaluate definite integrals of polynomial functions.
- Find values of integrals of polynomial functions.

Definite Integrals:

$$\int_a^b f(x)\,dx = f(b) - f(a)$$

<u>Example 1:</u>

Evaluate:

a. $\int_0^5 x^2 dx$

b. $\int_{-2}^3 (x^2 + 2x)dx$

c. $\int_{-1}^1 (x^3 - 2)dx$

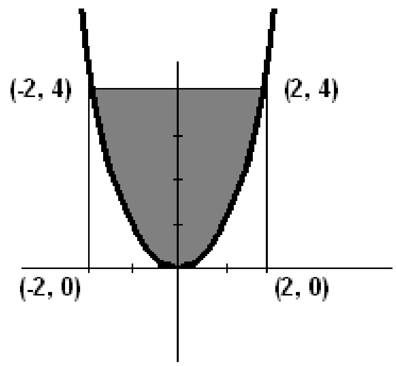
d. $\int_0^{24} (3x^2 + 3x + 3)dx$

15.12 Writing Integrals from Regions

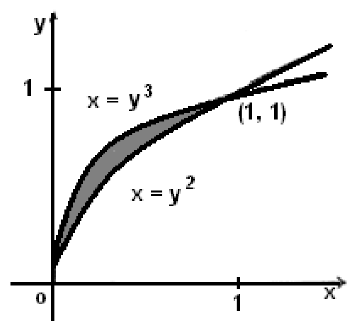
Objectives:

- Write and evaluate integrals from a given region

Example 1:
Find the area of the shaded region.



Example 2:
Find the area of the shaded region.



Example 3:
Find the area of the shaded region.