Ad. Math –Homework Packet
Chapter 15B – Integrals

NAME Key

15.9: Distance and Riemann Sums

1. What is the total distance traveled by a car, which travels at a rate of 65mph for 1.5 hours, 15mph for 30 minutes, and 40mph for 45 minutes?

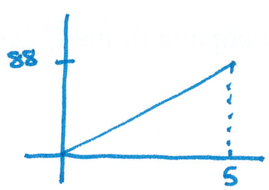
$d = rt$
$= 65(1.5) + 15(.5) + 40(.75)$
$= 97.5 + 7.5 + 30$
$= \boxed{135 \text{ miles}}$

2. You hear a commercial claiming a car can go from "0 to 60" in 5 seconds.

   a. How fast is 60 miles per hour in terms of feet per second? (It's in your notes!!)

   $\frac{60 \text{ mile}}{1 \text{ hr.}} \cdot \frac{1 \text{ hour}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} = \boxed{88 \text{ ft./sec.}}$

   b. Graph the situation above. Let the measurement of each "block" be .5 seconds by 10 ft/s.



   c. Assuming acceleration was constant, write a formula for the velocity of the car (in f/s). (It's a LINE, people)

   y-int = 0
   slope = $\frac{88}{5}$

   $\boxed{y = \frac{88}{5}x}$

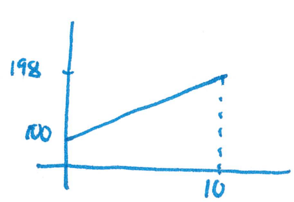
   d. How far did the car go in the first **5 seconds**? How far did it go in the first **8 seconds**?

   $A = \frac{1}{2}(5)88$
   $= \boxed{220 \text{ ft.}}$

   (8, 140.8)
   $A = \frac{1}{2}(8)(140.8)$
   $= \boxed{563.2 \text{ ft}}$

3. Suppose a space probe travels on a straight line with an initial speed of 100m/sec and a constant acceleration of 9.8 m/sec^2. Then its velocity at time t is given by 100 + 9.8t. Find the distance it will have traveled in 10 seconds.

$y = 100 + 9.8t$

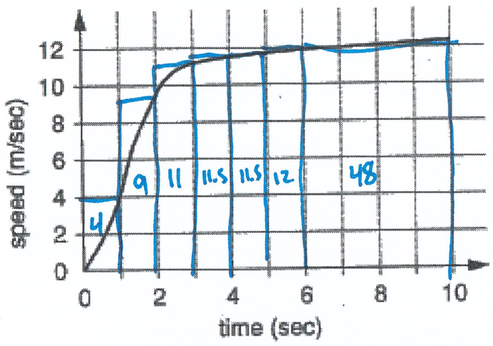


$A = \frac{1}{2}h(b_1 + b_2)$
$= \frac{1}{2}(10)(100 + 198)$
$= \boxed{1490 \text{ m}}$

For 4 and 5, each rate-time graph depicts a runner competing in a track event., From the graph, estimate the distance of the race. Use Riemanns!

4.

Sample: Right

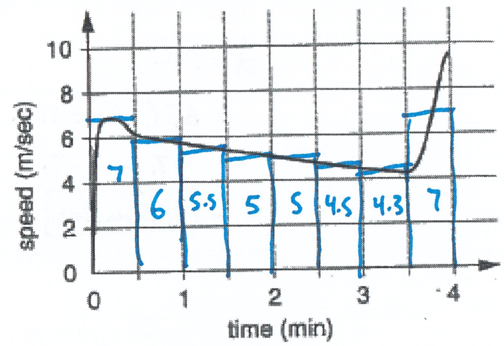


$4 + 9 + 11 + 11.5 + 11.5 + 12 + 4(12)$

$\approx \boxed{107m}$

5.

Sample: Middle



$\approx \boxed{44.3m}$

6. For the function $f(x) = 3x^2 - 1$, calculate the Riemann sum over the given interval. (Hint: Draw a picture.)

a. Over $0 \leq x \leq 2$ for $\Delta x = .25$, when z_i = the left endpoint of the i^{th} interval.

$\approx \boxed{4.5654}$

b. Over $0 \leq x \leq 8$ with 4 subintervals, when z_i = the right endpoint of the i^{th} interval

$\approx \boxed{712}$

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15.10: Area Under the Curve

Find each anti-derivative

1. $\int 4dx$ $4x + C$

2. $\int xdx$ $\frac{x^2}{2} + C$

3. $\int (2x+2)dx$ $\frac{2x^2}{2} + 2x + C = x^2 + 2x + C$

4. $\int (3x-6)dx$ $\frac{3x^2}{2} - 6x + C$

5. $\int (3x^3 - x^2 + 2)dx$ $\frac{3x^4}{4} - \frac{x^3}{3} + 2x + C$

6. $\int (x^{\frac{5}{4}} - x^{\frac{3}{4}} + 4x^{\frac{1}{2}})dx$

$$\frac{x^{9/4}}{\frac{9}{4}} - \frac{x^{\frac{7}{4}}}{\frac{7}{4}} + \frac{4x^{3/2}}{\frac{3}{2}} + C$$

$$= \frac{4x^{9/4}}{9} - \frac{4x^{7/4}}{7} + \frac{8x^{3/2}}{3} + C$$

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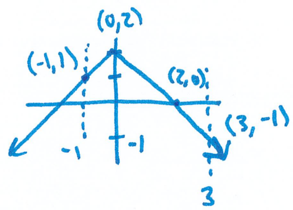
Evaluate the area under each curve by ***examining the graph of the function*** *.(DRAW A PICTURE!!)*

7. $\int_{-1}^{3}(-|x|+2)dx$



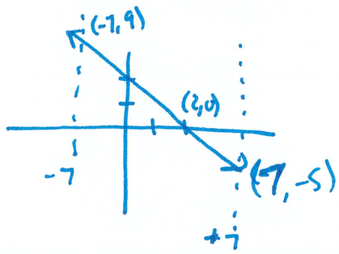
$\frac{1}{2}h(b_1+b_2) + \frac{1}{2}bh + \frac{1}{2}bh$

$\frac{1}{2}(1)(1+2) + \frac{1}{2}(2)(2) + \frac{1}{2}(1)(-1)$

$1.5 + 2 + (-.5)$

$\boxed{3 \text{ un.}^2}$

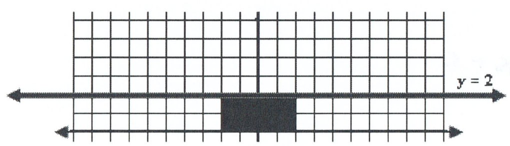
8. $\int_{-7}^{7}(2-x)dx$



$\frac{1}{2}(9)(9) + \frac{1}{2}(5)(-5)$

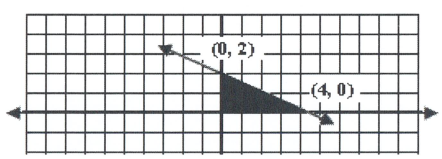
$40.5 - 12.5 = \boxed{28 \text{ un.}^2}$

9.



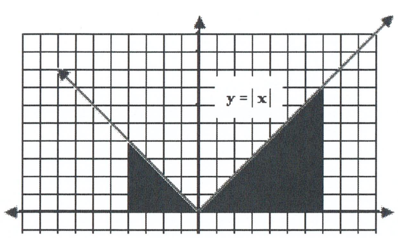
$4(2) = \boxed{8 \text{ un.}^2}$

10.



$\frac{1}{2}(4)(2) = \boxed{4 \text{ un.}^2}$

11.



$\frac{1}{2}(4)(4) + \frac{1}{2}(7)(7)$

$8 + 24.5$

$\boxed{32.5 \text{ un}^2}$

12. Graph the velocity function V = $t^2 + 4$ where $0 \le t \le 5$.

c. Use your calculator to graph the function between t = 0 and t = 5.

d. Determine the area under the curve between t = 0 and t = 5.

$\frac{185}{3}$ or $61\,2/3 \text{ un}^2$

e. How do we express this area with mathematical symbols?

$\int_0^5 t^2+4\,dx$

15.11: Basic Integration

Use the properties of integrals to write the expression as a single integral (if it is not already). Then, find the exact value of the integral.

1. $\int_2^5 4dx$

$4x \Big|_2^5 = 4(5) - 4(2) = \boxed{12}$

2. $\int_{-1}^3 xdx$

$\frac{x^2}{2} \Big|_{-1}^3 = \frac{(3)^2}{2} - \frac{(-1)^2}{2} = \boxed{4}$

3. $\int_{-2}^2 (2x+2)dx$

$x^2+2x \Big|_{-2}^2 = ((2)^2+2(2)) - ((-2)^2+2(-2)) = \boxed{8}$

4. $\int_3^7 3(x-2)dx$

$\frac{3x^2}{2} - 6x \Big|_3^7 = 31.5 - (-4.5) = \boxed{36}$

5. $\int_2^5 (2x+2)dx + 2\int_5^{10} (x+1)dx$

$\frac{2x^2}{2} + 2x \Big|_2^5 \; 35 - (8) = 27$

$\frac{x^2}{2} + x \Big|_5^{10} \; 2(60 - 17.5) = 85$

$27 + 85 = \boxed{112}$

6. $\int_{-1}^4 4dx + \int_4^5 4dx$

$4x \Big|_{-1}^4 \; 16 - (-4) = 20$

$4x \Big|_4^5 \; 20 - 16 = 4$

$20 + 4 = \boxed{24}$

7. $\int_0^5 (x^2+2)dx + 3\int_0^5 xdx$

$\frac{x^3}{3} + 2x \Big|_0^5 \; 51\frac{2}{3}$

$\frac{x^2}{2} \Big|_0^5 \; 3(12.5) = 37.5$

$51\frac{2}{3} + 37\frac{1}{2} = \boxed{89.1\overline{6}}$

8. $\int_0^{15} x^2dx - \int_{10}^{15} x^2dx$

$\frac{x^3}{3} \Big|_0^{15} \; 1125$

$\frac{x^3}{3} \Big|_{10}^{15} \; 1125 - 333\frac{1}{3} = 791\frac{2}{3}$

$1125 - 791\frac{2}{3} = \boxed{333\frac{1}{3}}$

9. $\int_{-3}^5 (x^2+4)dx$

$\frac{x^3}{3} + 4x \Big|_{-3}^5 = \left(\frac{5^3}{3} + 4(5)\right) - \left(\frac{(-3)^3}{3} + 4(-3)\right) =$

6x

$\boxed{82\frac{2}{3}}$

10. $\int_2^{10} (x^2+5x+2)dx$

$\frac{x^3}{3} + \frac{5x^2}{2} + 2x \Big|_2^{10} = \left(\frac{10^3}{3} + \frac{5(10)^2}{2} + 2(10)\right) - \left(\frac{2^3}{3} + \frac{5(2)^2}{2} + 2(2)\right)$

$= \boxed{\frac{1760}{3} \text{ or } 586\frac{2}{3}}$

15.12: Bounded Curves and Other Basic Integration

Write the definite integral represented in each. Then evaluate the integral.

1. $f(x) = -.1x^2 + 7$ from x = 0 to x = 5.

$$\int_0^5 (-.1x^2+7)\,dx = \frac{-.1x^3}{3} + 7x \Big|_0^5 = 30\tfrac{5}{6} - 0 = \boxed{30\ 5/6}$$

2. $f(x) = 3x^3 + 2x - 4$ from x = 3 to x = 11.

$$\int_3^{11} (3x^3+2x-4)\,dx = \frac{3x^4}{4} + x^2 - 4x \Big|_3^{11} = 11057.75 - 57.75 = \boxed{11{,}000}$$

3. $f(x) = 4$ from x = -3 to x = 3.

$$\int_{-3}^{3} 4\,dx = 4x \Big|_{-3}^{3} = 12 - (-12) = \boxed{24}$$

4. $f(x) = -\cos x$ from x = 0 to x = 1. (HINT: Think back to Chapter 13A…)

$$\int_0^1 -\cos x\,dx = -1\int_0^1 \cos x\,dx \quad \sin x \Big|_0^1 -1(0-1) = -1(-1) = \boxed{1}$$

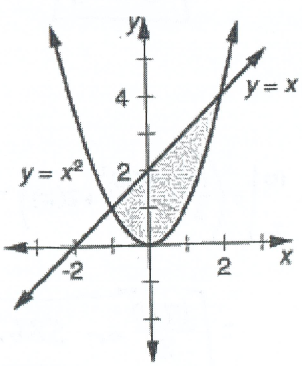
5. Suppose a car accelerates from 0 to 100 ft/sec in 5 seconds so that its velocity in ft/sec after t seconds is given $v(t) = .25(t-5)^2 + 100$. What is the total distance traveled in the 5 second interval?

$$\int_0^5 (.25t^2 - 2.5t + 6.25 + 100)\,dx$$

$$\int_0^5 (.25t^2 - 2.5t + 106.25)\,dx = \frac{t^3}{12} - \frac{2.5t^2}{2} + 106.25t \Big|_0^5 = \boxed{85\ 5/12}$$

For 5 and 6, express the area of the shaded region using integral notation and find its value.

6.



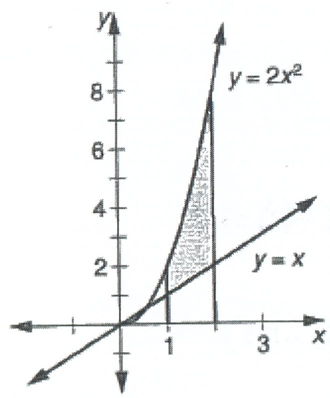
$$\int_{-1}^{2} (x+2)\,dx - \int_{-1}^{2} x^2\,dx$$

$$\frac{x^2}{2} + 2x \Big|_{-1}^{2}\ 6 - -1.5 = 7.5$$

$$\frac{x^3}{3} \Big|_{-1}^{2} = \frac{8}{3} + \frac{1}{3} = 3$$

$$7.5 - 3 = \boxed{4.5}$$

7.



$$\int_1^2 2x^2\,dx - \int_1^2 x\,dx$$

$$\frac{2x^3}{3} \Big|_1^2 \frac{16}{3} - \frac{2}{3} = \frac{14}{3}$$

$$\frac{x^2}{2} \Big|_1^2 2 - \frac{1}{2} = \frac{3}{2}$$

$$\boxed{3\tfrac{1}{6}}$$