Ch 15 A Derivatives TEST REVIEW

Name Key

Advanced Math

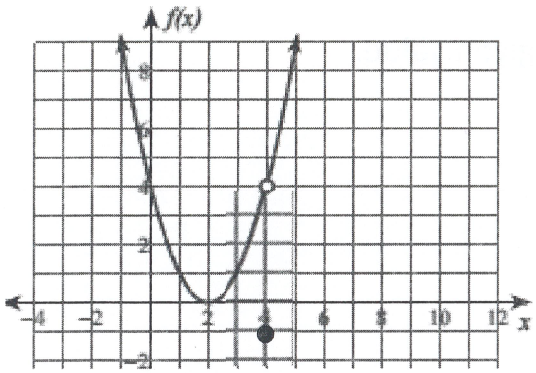
Evaluate the following.

1. $\lim_{x\to -3} \frac{x-3}{x^2+2x+2} = \frac{-6}{5}$

2. $\lim_{x\to -4} \frac{\cancel{x+4}}{\underbrace{x^2+5x+4}_{(x+1)\cancel{(x+4)}}} = \frac{1}{-3}$

3. $\lim_{x\to 2} \frac{\overbrace{x^2-4}^{(x+2)\cancel{(x-2)}}}{\cancel{x-2}} = 4$

4. Use the graph below to evaluate the following. Find f(4) = -1



$\lim_{x\to 4} f(x) = 4$

5. Use the **limit definition** of the derivative to find the derivative of each function with respect to x.

a. $y = 4x^2 + 4$

$f(x+h) = 4(x+h)^2 + 4$

$= 4x^2 + 8xh + 4h^2 + 4$

$\lim_{h\to 0} \frac{4x^2+8xh+4h^2+4-4x^2-4}{h}$

$\lim_{h\to 0} \frac{8xh+4h^2}{h}$

$\lim_{h\to 0} \frac{\cancel{h}(8x+4h)}{\cancel{h}}$

$\lim_{h\to 0} 8x+4h$

$\boxed{f'(x) = 8x}$

b. $f(x) = 4x^2 + 4x - 3$

$f(x+h) = 4(x+h)^2 + 4(x+h) - 3$

$= 4x^2 + 8xh + 4h^2 + 4x + 4h - 3$

$\lim_{h\to 0} \frac{4x^2+8xh+4h^2+4x+4h-3-4x^2-4x+3}{h}$

$\lim_{h\to 0} \frac{8xh+4h^2+4h}{h}$

$\lim_{h\to 0} \frac{h(8x+4h+4)}{h}$

$\lim_{h\to 0} 8x+4h+4$

$\boxed{f'(x) = 8x+4}$

6. a. Find the average rate of change for the function $y=-x^2+x+2$ at -2 and 1.

$(-2,-4)$ $(1,2)$

$\frac{2-(-4)}{1-(-2)} = \frac{6}{3} = \boxed{2}$

b. Find the instantaneous velocity at x = 3.

$f'(x) = -2x+1$

$f'(3) = -2(3)+1 = \boxed{-5}$

c. Find the instantaneous acceleration at x = 3.

$f''(x) = -2$

$f''(3) = \boxed{-2}$

Find the equation of the tangent line of the function at the given value.

a) $y=x^3-3x^2+2$ at x = 3.

$y = (3)^3-3(3)^2+2$
$= 2$

$(3,2)$

$f'(x) = 3x^2-6x$

$f'(3) = 3(3)^2-6(3)$
$= 9$

$\boxed{y-2 = 9(x-3)}$

b) $y=-\frac{5}{x^2+1}$ at x = -2.

$y = \frac{-5}{(-2)^2+1}$
$= -1$

$(-2,-1)$

$f'(x) = \frac{(x^2+1)(0)-(-5)(2x)}{(x^2+1)^2}$

$f'(-2) = \frac{10(-2)}{((-2)^2+1)^2} = \frac{-20}{25} = -\frac{4}{5}$

$\boxed{y+1 = -\frac{4}{5}(x+2)}$

Find the derivative of the following.

a) $y = 5x^7 + 4x$

$\boxed{y' = 35x^6 + 4}$

b) $y = -2x^3 - 4x^{-3}$

$y' = -6x^2 + 12x^{-4}$

$\boxed{y' = -6x^2 + \frac{12}{x^4}}$

c) $y = \frac{5}{4}x^{\frac{2}{3}}$

$y' = \frac{\frac{5}{6}}{x^{1/3}}$

$y' = \frac{5}{6\sqrt[3]{x}} \cdot \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \boxed{\frac{5\sqrt[3]{x^2}}{6x}}$

d) $y = \frac{2}{2x^4 - 5}$

$y' = \frac{(2x^4-5)(0) - 2(8x^3)}{(2x^4-5)^2}$

$\boxed{y' = \frac{-16x^3}{(2x^4-5)^2}}$

e) $y = (-2x^4 + 5x^2 + 4)(-3x^2 + 2)$

$y' = (-2x^4+5x^2+4)(-6x) + (-3x^2+2)(-8x^3+10x)$

$y' = 12x^5 - 30x^3 - 24x + 24x^5 - 30x^3 - 16x^3 + 20x$

$\boxed{y' = 36x^5 - 76x^3 - 4x}$

f) $y = \frac{3\sin x}{(2x + 5)}$

$y' = \frac{(2x+5)(3\cos x) - 3\sin x\,(2)}{(2x+5)^2}$

$\boxed{y' = \frac{6x\cos x + 15\cos x - 6\sin x}{(2x+5)^2}}$

g) $y = (5x^2 + 3)^4$

$u = 5x^2+3$ $\quad f(u) = u^4$

$u' = 10x$ $\quad f'(u) = 4u^3$

$y' = 10x \cdot 4u^3$

$\boxed{y' = 40x(5x^2+3)^3}$

h) $f(x) = \sin(2x^3)$

$u = 2x^3$ $\quad f(u) = \sin u$

$u' = 6x^2$ $\quad f'(u) = \cos u$

$\boxed{f'(x) = 6x^2 \cdot \cos(2x^3)}$

i) $y = \ln x^3$

$u = x^3$ $\quad f(u) = \ln u$

$u' = 3x^2$ $\quad f'(u) = \frac{1}{u}$

$y' = \frac{3x^2}{u}$

$y' = \frac{3x^2}{x^3}$

$\boxed{y' = \frac{3}{x}}$

j) $f(x) = \sqrt{-2x^2 + 1}$

$u = -2x^2+1$ $\quad f(u) = u^{1/2}$

$u' = -4x$ $\quad f'(u) = \frac{1}{2}u^{-1/2}$

$= \frac{1}{2\sqrt{u}} \cdot \frac{\sqrt{u}}{\sqrt{u}} = \frac{\sqrt{u}}{2u}$

$f'(x) = -4x \cdot \frac{\sqrt{u}}{2u}$

$= \frac{-4x\sqrt{-2x^2+1}}{-4x^2+2} = \boxed{\frac{-2x\sqrt{-2x^2+1}}{-2x^2+1}}$