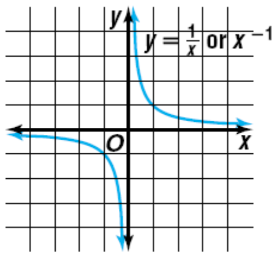
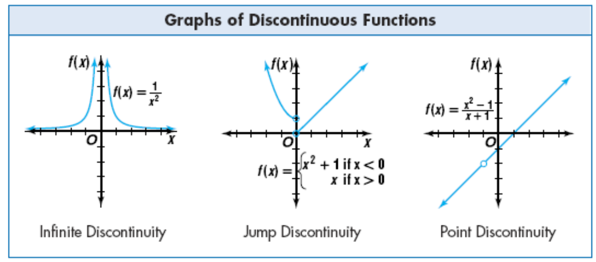
15.1 - Limits

Review:
Recall that during first semester, while graphing rational functions, we learned how to take a limit "as x approaches infinity" and discussed what it meant to be "continuous". This chapter, we will continue this discussion (better late than never…right?)

rational function





Find each limit:

a. $\lim\limits_{x\to\infty} \dfrac{3x^2 + 4x - 1}{2x}$

b. $\lim\limits_{x\to\infty} \dfrac{x^2}{x^3}$

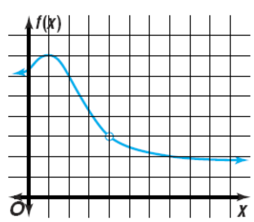
c. $\lim\limits_{x\to\infty} \dfrac{n}{n + 1}$

d. $\lim\limits_{x\to\infty} \dfrac{20x^2 + 10x + 6}{4x^2 - 30n + 2}$

Finding limits "as x approaches a number"

<u>Example 1:</u>
Consider the graph of the function y = f(x) shown below. Find each pair of values.

a. f(2) and $\lim\limits_{x\to 2} f(x)$



b. f(4) and $\lim\limits_{x\to 4} f(x)$

<u>Example 2:</u>
Evaluate each limit:

a. $\lim\limits_{x\to 3}(x^3 - 5x^2 + 7x - 10)$

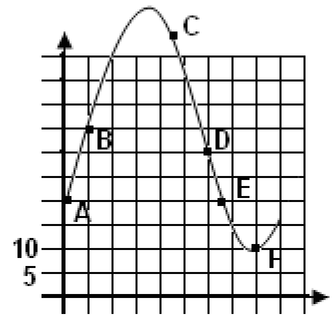
b. $\lim\limits_{x\to \pi} \dfrac{\cos(x)}{x}$

c. $\lim\limits_{x\to 4} \dfrac{x^2 - 2x - 8}{x^2 - 4x}$

Ad. Math Notes
Chapter 15A – Derivatives

15.2 - Average Rate of Change

In 1 – 5, refer to the graph at the right.



1. In going from B to C, find Δx and Δy.

2. Between which two consecutive points is Δy = 15?

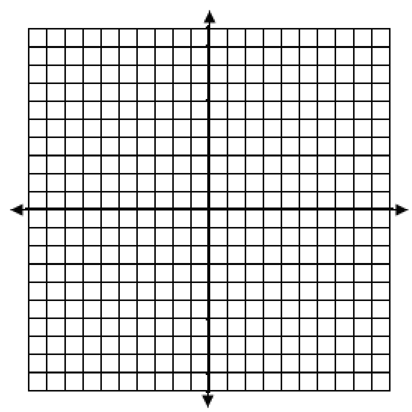
3. Between which two consecutive points is Δy = -10?

4. Find the *average rate of change* of the function from x = 0 to x = 3.

5. Between which two points was the *average rate of change* of the function zero?

6. A rocket is propelled vertically into the air from a height of 20 ft with an initial velocity of 480 ft/sec. If only the effect of gravity is considered, then its height (in feet) after t seconds is given by the equation $h(t) = 480t - 16t^2 + 20$.

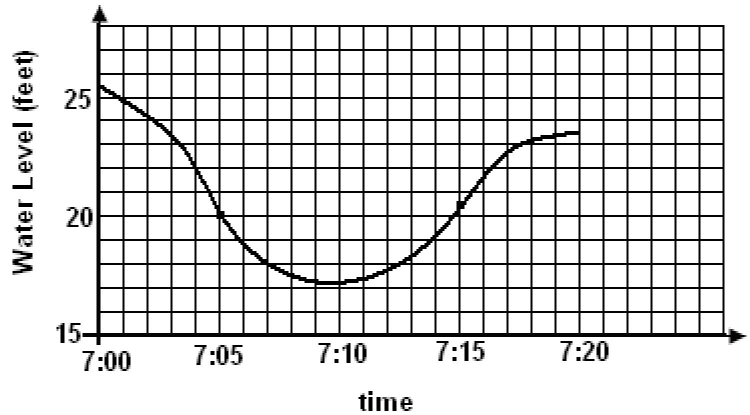
   a. Sketch a graph of height vs. time.
   (Use your calculator to help if you don't remember)



   b. Find the *average velocity (rate of change)* from t = 4 to t = 12. Include appropriate units with your answer.

   c. Find the approximate slope of the **tangent line** of t = 5 by sketching the line.

7. The water level w(t) in a reservoir at various times t is graphed at the right. Estimate the *average rate of change* of *w* at 7:05 and at 7:15



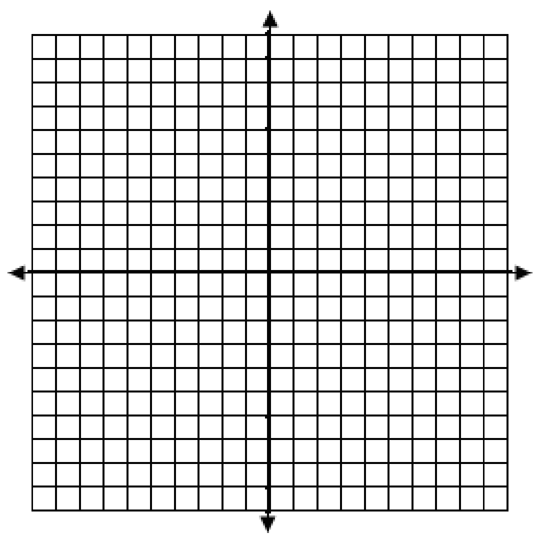
Give three expressions equal to $\frac{\Delta y}{\Delta x}$.

8. A projectile is propelled into the air from ground level with an initial velocity of 800 ft/sec. If only the effect of the Earth's gravity is considered, its height (in feet) after t seconds is given by the function $h(t) = 800t - 16t^2$. Find the *average velocity* over the following intervals.

a. $10 \leq x \leq 20$

b. $20 \leq x \leq 30$

c. $30 \leq x \leq 40$



d. What is the *instantaneous velocity* of the projectile at t = 5 seconds?

15.3 - Limit Definition for Derivatives

1. Write the Limit Definition of the Derivative of the function f(x).

Use the Limit Definition to algebraically calculate the derivative of each function below.

2. $f(x) = 17x – 3$

3. $f(x) = 8x^2$

ACTIVITY

PART A: Complete the following on your own, all step-by-step work MUST be shown neatly and sequentially on another sheet of paper. Record the answers on the blanks below. (Also record the answers from *2* and *3*)

1. $f(x) = 17x - 3$ ____________________________

2. $f(x) = 8x^2$ ____________________________

3. $f(x) = x^2 + 5$ ____________________________

4. $f(x) = 3x^2 - 4x$ ____________________________

5. $f(x) = 4x^3 + 9$ ____________________________

6. $f(x) = 9x^2 - 13x + 7$ ____________________________

7. $f(x) = x^4$ ____________________________

Go back and review your answers to questions 1 - 7. Compare the answers of $f'(x)$ to their functions $f(x)$. Can you see a relationship between the function and its derivative? There is a *"trick"* to finding the derivative of a polynomial function….can you figure out what it is? Discuss your idea with other students and then check your idea(s) with the teacher BEFORE completing #8.

8. Use the quick "trick" to find the derivative of the following functions.

   a. $f(x) = 3x^5 + 8x^3 - 6x^2 + 9$ ____________________________________

   b. $f(x) = -23x^7 - 6x$ ____________________________________

   c. $f(x) = 8x + 3$ ____________________________________

   d. $f(x) = 35$ ____________________________________

15.4 - Velocity and Acceleration

Recall that average velocity is merely the slope between two points.

Find the average velocity of S(t) = $t^2 - 16$ *from t = 2*

1. *t* = 6

2. *t* = 3

3. *t* = 2.2

4. *t = 2.1*

For problems 4-9, let the position function of a moving particle be S(t)= $4t^2 - t + 1$.

5. Determine a formula for the <u>average</u> velocity.

6. Use this formula to find the average velocity on the interval [1, 3].

7. Determine a formula for the instantaneous velocity.

8. What is the difference between *average* and *instantaneous*?

9. Determine a formula for the instantaneous acceleration.

10. Let $f(x) = x^2 + 4$ for all real numbers x. Find f ’(3), *the derivative of f at x = 3*

11. So what does a *derivative* actually give you?

15.5 - Writing Tangent Lines

To write an equation of a line, we need to know 2 things: a _______________ and a _____________.

Derivatives are _________________.

Write an equation for the tangent line in each given situation. Leave your answer in ***point-slope*** *form!*

1. *f(x)*= $2x+3$ at *x*=1.

Point: ______________

Slope @ x = 1: __________________

Tangent line equation: ________________________

2. *f(x)*= $\frac{20}{x^2}$ at *x*=-2.

3. *f(x)*= $x^{\frac{4}{3}}$ at *x*=8.

Ad. Math
Chapter 15 Notes

15.6 - Product Rule

Function	Derivative
e^x	
$\ln x$	
$\sin x$	
$\cos x$	

Use the product rule to find the derivative of the function described by the given equation. Write the answer in a simplified form.

PRODUCT RULE:

1. $f(x) = (x+1)(x^3 - z^2 + 10)$

2. $f(x) = (e^x + 2)(x^2 - x)$

3. $f(x) = (\sin x)(2x + 7)$

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Chapter 15 Notes

15.7 - Quotient Rule

Use the quotient rule to find the derivative of each of the function listed below.

QUOTIENT RULE:

1. $f(x) = \frac{3x^2}{x+2}$

2. $f(x) = \frac{2x^2 + 3x + 1}{x}$

3. $f(x) = \frac{\ln x}{x}$

4. $f(x) = \frac{xe^x}{x^2 + 2}$

5. Find the slope of the line tangent to the function at the given point: $f(x) = \frac{1}{x-2}$ at x=4

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Chapter 15 Notes

15.8 - Chain Rule

Use the chain rule to find the derivative of each function.

CHAIN RULE:

1. *f(x)*= $(2x+1)^5$

2. *f(x)*= $\sin(3x)$

3. *f(x)*= $\ln(4x^2+3x+9)$

4. *f(x)*= $e^{(x^2+x)}$