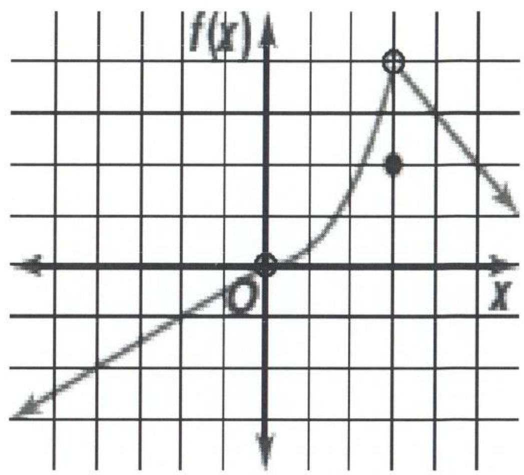
Ad. Math –SET PACKET
Chapter 15A – Derivatives

NAME Key

SET 1: Limits

1. Use the graph of y = f(x) to find each pair of values.



a. f(-2) and $\lim_{x\to -2} f(x)$ = -1

-1

b. b. f(0) and $\lim_{x\to 0} f(x)$ = 0

undefined

c. c. $\lim_{x\to 3} f(x)$ = 4 and f(3) = 2

2. Evaluate each limit:

a. $\lim_{x\to 2}(-4x^2 - 3x + 6) = -16$

b. $\lim_{x\to \pi} \frac{\sin x}{x} = 0$

c. $\lim_{x\to -1}(-x^3 + 3x^2 - 4) = 0$

d. $\lim_{x\to 0}(x + \cos x) = 1$

3. Evaluate each limit:

a. $\lim_{x\to 5} \frac{x^2 - 25}{x - 5} = \boxed{10}$

$\frac{(x-5)(x+5)}{(x-5)}$

b. $\lim_{x\to 0} \frac{2n^2}{n} = \boxed{0}$

$\frac{2n}{1}$

c. $\lim_{x\to 3} \frac{x^2 - 3x}{x^2 + 2x - 15} = \boxed{\frac{3}{8}}$

$\frac{x(x-3)}{(x+5)(x-3)} = \frac{x}{x+5}$

d. $\lim_{x\to 0} \frac{x^3 - x^2 + 2x}{x^3 + 4x^2 - 2x} = \boxed{-1}$

$\frac{x(x^2-x+2)}{x(x^2+4x-2)} = \frac{(x^2-x+2)}{(x^2+4x-2)}$

e. $\lim_{x\to 0} \frac{x \cos x}{x^2 + x} = \boxed{1}$

$\frac{x \cos x}{x(x+1)} = \frac{\cos x}{x+1}$

f. $\lim_{x\to 0} \frac{(x + 2)^2 - 4}{x} = \boxed{4}$

$\frac{x^2+4x+4-4}{x} = \frac{x(x+4)}{x} = x+4$

SET 2: Secant Lines

1. Using $x^3 - 6x + 2$, find each Average Rate of Change between each pair of constant points of the function. (You may want to generate an x-y table to make the process go more quickly).

Constant Points	**A.R.C. work**	Average Rate of Change
-8 to 4	$\frac{42-(-462)}{4-(-8)} = \frac{504}{12}$	42
-6 to 2	$\frac{-2-(-178)}{2-(-6)} = \frac{176}{8}$	22
-4 to 0	$\frac{2-(-38)}{0-(-4)} = \frac{40}{4}$	10
-3 to –1	$\frac{7-(-7)}{-1-(-3)} = \frac{14}{2}$	7
-2.5 to –1.5 1.375 7.625	$\frac{7.625-1.375}{-1.5-(-2.5)} = \frac{6.25}{1}$	6.25
-2.25 to –1.75 4.109 7.141	$\frac{7.141-4.109}{-1.75-(-2.25)} = \frac{3.032}{.5}$	6.064
-2.1 to –1.9 5.339 6.541	$\frac{6.541-5.339}{-1.9-(-2.1)} = \frac{1.202}{.2}$	6.01
-2.05 to –1.95 5.685 6.285	$\frac{6.285-5.685}{-1.95-(-2.05)} = \frac{.6}{.1}$	6
-2.01 to –1.99 5.939 6.059	$\frac{6.059-5.939}{-1.99-(-2.01)} = \frac{.12}{.02}$	6

2. Using graph paper, carefully graph the function and *lightly* graph the Secant Line associated with each A.R.C.

3. What does your graph tell you about Instantaneous Rate of Change and a possible Tangent Line?

@ $x = -2$
Instantaneous Rate of Change = 6

@ $(-2, 6)$
$m = 6$

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ET 3: Polynomial Derivative "Tricks"

Use the Polynomial tricks to find f'(x) for each f(x).

1. $f(x) = x^7$ $\quad f'(x) = 7x^6$

2. $f(x) = 13x^4$ $\quad f'(x) = 52x^3$

3. $f(x) = 4$ $\quad f'(x) = 0$

4. $f(x) = 3x^2 + 2$ $\quad f'(x) = 6x$

5. $f(x) = \dfrac{1}{x^{101}}$ $\quad f(x) = x^{-101}$ $\quad f'(x) = \dfrac{-101}{x^{102}}$

6. $f(x) = \dfrac{7}{x^5}$ $\quad f(x) = 7x^{-5}$ $\quad f'(x) = \dfrac{-35}{x^6}$

$f(x) = 5x^{10} + x^5 - 2x^2 + 1$ $\quad f'(x) = 50x^9 + 5x^4 - 4x$

8. $f(x) = x^5 - x^4 + x^2 - x + 1$ $\quad f'(x) = 5x^4 - 4x^3 + 2x - 1$

9. $f(x) = 12x - 3$ $\quad f'(x) = 12$

10. $f(x) = \dfrac{1}{x^4} - \dfrac{2}{x^2} + x$ $\quad f(x) = x^{-4} - 2x^{-2} + x$ $\quad f'(x) = \dfrac{-4}{x^5} + \dfrac{4}{x^3} + 1$

Find the slope of the line tangent to each function at x=2.

11. $f(x) = x^7$ $\quad f'(x) = 7x^6$ $\quad f'(2) = 448$

12. $f(x) = x - 2$ $\quad f'(x) = 1$ $\quad f'(2) = 1$

13. $f(x) = x^2 + 2x - 3$ $\quad f'(x) = 2x + 2$ $\quad f'(2) = 6$

14. $f(x) = x^{\frac{1}{2}}$ $\quad f'(x) = \frac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{\sqrt{x}}{2x}$ $\quad f'(2) = \dfrac{\sqrt{2}}{4}$

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SET 4: Average Rate of Change (The Basics)

For problems 1-3, let the position function of a moving particle be S(t)= $t^2 - 25$. ft/sec.
Find the $\boxed{\textit{average velocity}}$ *from t=3 to*
(3,-16)

1. t=5

(5,0) $m = 8$ ft./sec.

2. t=4

(4,-9) $m = 7$ ft./sec.

3. t=3.1

(3.1, -15.39) $m = 6.1$ ft./sec

For problems 4-9, let the position function of a moving particle be S(t)= $3t^2 - 2t + 1$.

4. Determine a <u>formula</u> for the <u>average</u> velocity (the average rate of change).

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta Y}{\Delta X}$$

$$\boxed{6t + 3\Delta t - 2}$$

5. Use this formula to find the average velocity on the interval [0,2] which means $0 \le x \le 2$.

(0,1)
(2,9)

$\frac{9-1}{2-0} = 4$ ft./sec.

6. Determine a formula for the instantaneous velocity, also known as $f'(\cancel{x})$.

$$\lim_{\Delta t \to 0} 6t + 3\Delta t - 2 = \boxed{6t - 2}$$

7. Use this formula to find the instantaneous velocity at t=0.

$f'(0) = -2$ ft./sec.

8. Determine a formula for the instantaneous acceleration, also know as $f''(\cancel{x})$.

$f''(\cancel{x}) = 6$

9. Use this formula to find the instantaneous acceleration at t=0.

$f''(0) = 6$ ft./sec.

SET 5: Velocity and Acceleration

For each position function, find v(t) and a(t).

$f'(x)$ $f''(x)$

	Position Function	Velocity Function	Acceleration Function
1)	$S(t) = 2t^2 + 5t - 12$	$s'(t) = 4t + 5$	$s''(t) = 4$
2)	$S(t) = 4t + 3$	$s'(t) = 4$	$s''(t) = 0$
3)	$S(t) = 4 - 2t - t^2$	$s'(t) = -2 - 2t$	$s''(t) = -2$
4)	$S(t) = (2t+3)^2$ $= 4t^2 + 12t + 9$	$s'(t) = 8t + 12$	$s''(t) = 8$
5)	$S(t) = (4-t)^2$ $= 16 - 8t + t^2$	$s'(t) = -8 + 2t$	$s''(t) = 2$
6)	$S(t) = 64t - 16t^2$	$s'(t) = 64 - 32t$	$s''(t) = -32$
7)	$S(t) = 2t^3 - 18t^2 + 48t - 6$	$s'(t) = 6t^2 - 36t + 48$	$s''(t) = 12t - 36$
8)	$S(t) = 2t^3 - 9t^2 + 12$	$s'(t) = 6t^2 - 18t$	$s''(t) = 12t - 18$

Use the following information for problems 9 - 11.

If a ball is thrown vertically upward with a velocity of 32 feet per second, the ball's height after t seconds is given by $S(t) = 32t - 16t^2$. $S(t) = -16t^2 + 32t$

9. What time t does the ball reach its maximum height?

$s'(t) = -32t + 32$
$0 = -32t + 32$
$t = 1$

1 second

10. What is the maximum height? $S(1) = -16(1)^2 + 32(1)$
$= 16$

16 ft.

11. At what time does the ball hit the ground? $0 = -16t^2 + 32t$
$= -16t(t + -2)$
$t = 0$ $t - 2 = 0$
$\boxed{t = 2}$

2 seconds

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T 5 (continued)

Use the following information for problems 12 – 19..

If a ball is thrown vertically upward with an initial velocity of 128 feet per second, the ball's height after t seconds is given by $S(t) = 128t - 16t^2$.

12. What is the velocity function?

$s'(t) = 128 - 32t$

13. What is the velocity when t=

a. 2 seconds $s'(2) = 64$ ft/sec.

b. 4 seconds $s'(4) = 0$ ft./sec.

c. 6 seconds $s'(6) = -64$ ft./sec.

14. At what time is the velocity…

t?

a. 48 feet per second? $128 - 32t = 48$ $\boxed{t = 2.5 \text{ sec.}}$

b. 16 feet per second? $128 - 32t = 16$ $\boxed{t = 3.5 \text{ sec}}$

c. -48 feet per second? $128 - 32t = -48$ $\boxed{t = 5.5 \text{ sec.}}$

15. When is the velocity zero?

$128 - 32t = 0$ $\boxed{t = 4 \text{ sec.}}$

16. What is the height of the ball when the velocity is zero?

position $t=4$

$s(4) = 128(4) - 16(4)^2 = \boxed{256 \text{ ft.}}$

17. Is this the maximum height?

Yes

18. When does the ball hit the ground?

8 seconds

9. What is its velocity at that time?

$s'(8) = 128 - 32(8) = \boxed{-128 \text{ ft./sec.}}$

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SET 6: Writing Tangent Lines

Determine the equation of the line tangent to the given function f(x) at the point of tangency for the given x value. Leave your equation in point-slope form!

1. $f(x)=\frac{1}{2}x^2+2x$ at $x=3$.

$f'(x) = x+2$ $\quad f(3) = \frac{1}{2}(3)^2+2(3)$

$f'(3) = 5$ $\quad = 10.5$

$m=5$ $\quad (3, 10.5)$

$\boxed{y-10.5 = 5(x-3)}$

4. $f(x)= x^3-x$ at $x=-1$.

$f'(x) = 3x^2-1$ $\quad f(-1) = (-1)^3-(-1)$

$f'(-1) = 2$ $\quad = 0$

$m=2$ $\quad (-1,0)$

$\boxed{y = 2(x+1)}$

2. $f(x)=\frac{1}{2}x^2+2x$ at $x=1$.

$f'(x) = x+2$ $\quad f(1) = \frac{1}{2}(1)^2+2(1)$

$f'(1) = 3$ $\quad f(1) = 2.5$

$m=3$ $\quad (1,2.5)$

$\boxed{y-2.5 = 3(x-1)}$

5. $f(x)=\frac{1}{x^{-2}}$ at $x=3$.

$f(x) = x^2$ $\quad f(3) = (3)^2$

$f'(x) = 2x$ $\quad = 9$

$f'(3) = 6$ $\quad (3,9)$

$m=6$

$\boxed{y-9 = 6(x-3)}$

3. $f(x)= x^{\frac{1}{2}}$ at $x=4$.

$f'(x) = \frac{1}{2}x^{-1/2}$ $\quad f(4) = 4^{1/2}$

$= \frac{1}{2\sqrt{x}}$ $\quad = 2$

$= \frac{\sqrt{x}}{2x}$ $\quad (4,2)$

$f'(4) = \frac{1}{4}$

$m=\frac{1}{4}$

$\boxed{y-2 = \frac{1}{4}(x-4)}$

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SET 7: Product Rule

Function	Derivative
e^x	e^x
$\ln x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$

Use the product rule to find the derivative of the function described by the given equation. Write the answer in a simplified form.

1. $f(x) = x(x^3 - x^2 + 10)$

$f'(x) = 1(x^3 - x^2 + 10) + x(3x^2 - 2x)$

$f'(x) = x^3 - x^2 + 10 + 3x^3 - 2x^2$

$f'(x) = 4x^3 - 3x^2 + 10$

4. $f(x) = (x^2 - x^4)e^x$

$f'(x) = (2x - 4x^3)(e^x) + (x^2 - x^4)(e^x)$

$f'(x) = e^x(-x^4 - 4x^3 + x^2 + 2x)$

$f'(x) = xe^x(-x^3 - 4x^2 + x + 2)$

2. $f(x) = (x^2 + x)(x - 3)$

$f'(x) = (2x+1)(x-3) + (x^2+x)(1)$

$f'(x) = (2x^2 - 5x - 3) + (x^2 + x)$

$f'(x) = 3x^2 - 4x - 3$

5. $f(x) = (\ln x)(e^x - 7x)$

$f'(x) = \frac{1}{x}(e^x - 7x) + \ln x(e^x - 7)$

$f'(x) = \frac{e^x - 7x}{x} + \ln x(e^x - 7)$

$f'(x) = \frac{e^x}{x} - 7 + e^x \ln x - 7\ln x$

3. $f(x) = (\cos x)(x^2 - 7x)$

$f'(x) = -\sin x(x^2 - 7x) + \cos x(2x - 7)$

$f'(x) = -x^2\sin x + 7x\sin x + 2x\cos x - 7\cos x$

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SET 8: Quotient Rule

Use the quotient rule to find the derivative of each of the function listed below.

1. $f(x) = \dfrac{4x+2}{3x^2}$

$$f'(x) = \frac{(3x^2)(4) - (4x+2)(6x)}{(3x^2)^2}$$

$$= \frac{12x^2 - 24x^2 - 12x}{9x^4}$$

$$= \frac{-12x^2 - 12x}{9x^4} = \frac{-12x(x+1)}{9x^4} = \boxed{\frac{-4(x+1)}{3x^3}}$$

3. $f(x) = \dfrac{e^x}{2x}$

$$f'(x) = \frac{(2x)(e^x) - (e^x)(2)}{(2x)^2}$$

$$= \frac{2xe^x - 2e^x}{4x^2}$$

$$= \frac{2e^x(x-1)}{4x^2} = \boxed{\frac{e^x(x-1)}{2x^2}}$$

2. $f(x) = \dfrac{x}{x^2+2x+1}$

$$f'(x) = \frac{(x^2+2x+1)(1) - (x)(2x+2)}{(x^2+2x+1)^2}$$

$$= \frac{x^2+2x+1-2x^2-2x}{(x^2+2x+1)^2}$$

$$= \frac{-x^2+1}{(x^2+2x+1)^2}$$

$$= \frac{-(x^2-1)}{((x+1)^2)^2} = \frac{-(x+1)(x-1)}{(x+1)^4} = \boxed{\frac{-(x-1)}{(x+1)^3}}$$

4. $f(x) = \dfrac{x^2-2x}{e^x}$

$$f'(x) = \frac{(e^x)(2x-2) - (x^2-2x)(e^x)}{(e^x)^2}$$

$$= \frac{2xe^x - 2e^x - x^2e^x + 2xe^x}{e^{2x}}$$

$$= \frac{4xe^x - 2e^x - x^2e^x}{e^{2x}}$$

$$= \frac{e^x(4x-2-x^2)}{(e^x)^2} = \boxed{\frac{-x^2+4x-2}{e^x}}$$

5. Find the slope of the line tangent to the function at the given point: $f(x) = \dfrac{\ln x}{x-1}$ at x=3

$$f'(x) = \frac{(x-1)(\frac{1}{x}) - (\ln x)(1)}{(x-1)^2}$$

$$= \frac{\frac{x-1}{x} - \ln x}{x^2-2x+1}$$

$$= \boxed{\frac{1-\frac{1}{x} - \ln x}{(x-1)^2}}$$

$$f'(3) = \frac{\frac{3-1}{3} - \ln 3}{3^2-2(3)+1} = \frac{1-\frac{1}{3}-\ln 3}{4}$$

$$\approx -.10799$$

6. Find the coordinates of the points on the graph of $f(x)$ at which $f'(x) = 0$: $f(x) = \dfrac{x}{1+x^2}$

$$f'(x) = \frac{(1+x^2)(1) - x(2x)}{(1+x^2)^2}$$

$$= \frac{1+x^2-2x^2}{x^4+2x^2+1}$$

$$= \frac{-x^2+1}{x^4+2x^2+1}$$

$$\frac{-x^2+1}{x^4+2x^2+1} = 0$$

$$-x^2+1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

$$\boxed{(1, .5) \quad (-1, -.5)}$$

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SET 9: Product AND Quotient Rule

Caution: The functions below involve using the product rule along with the quotient rule.

1. $f(x)= \dfrac{xe^x}{x+1}$

$$f'(x) = \frac{(x+1)((x)(e^x)+e^x(1)) - (xe^x)(1)}{(x+1)^2}$$

$$= \frac{(x+1)(xe^x+e^x) - xe^x}{(x+1)^2}$$

$$= \frac{x^2e^x + 2xe^x + e^x - xe^x}{(x+1)^2} = \frac{x^2e^x + xe^x + e^x}{x^2+2x+1} = \boxed{\frac{e^x(x^2+x+1)}{(x+1)^2}}$$

2. $f(x)= \dfrac{e^x \ln x}{2-x}$

$$f'(x) = \frac{(2-x)((e^x)(\frac{1}{x}) + \ln x(e^x)) - (e^x \ln x)(-1)}{(2-x)^2}$$

$$= \frac{(2-x)(\frac{e^x}{x} + \ln x\, e^x) + e^x \ln x}{(2-x)^2}$$

$$= \frac{\frac{2e^x}{x} + 2\ln x e^x - \frac{xe^x}{x} - x\ln x e^x + e^x \ln x}{x^2-4x+4}$$

$$= \boxed{\frac{e^x(\frac{2}{x} + 3\ln x - 1 - x\ln x)}{(2-x)^2}}$$

3. $f(x)= \dfrac{x-1}{xe^x}$

$$f'(x) = \frac{(xe^x)(1) - ((x-1)(xe^x + e^x(1))}{(xe^x)^2}$$

$$= \frac{xe^x - (x^2e^x + xe^x - xe^x - e^x)}{(xe^x)^2}$$

$$= \frac{xe^x - (x^2e^x - e^x)}{(xe^x)^2}$$

$$= \frac{xe^x - x^2e^x + e^x}{(xe^x)^2} = \frac{e^x(x - x^2 + 1)}{x^2(e^x)^2}$$

$$= \boxed{\frac{-x^2+x+1}{x^2e^x}}$$

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SET 10: Chain Rule

Use the chain rule to find the derivative of each function.

1. $f(x) = (x^2+1)^5$

$u = x^2+1$

$f'(u) = 5u^4$

$f'(x) = 2x(5u^4) = 10x(x^2+1)^4$

2. $f(x) = \cos(4x)$

$u = 4x$

$f'(u) = -\sin(u)$

$f'(x) = 4(-\sin(4x))$

$= -4\sin(4x)$

3. $f(x) = e^{5x}$

$u = 5x$

$f'(u) = e^u$

$f'(x) = 5(e^{5x})$

$= 5e^{5x}$

4. $f(x) = (x^3-x)^{\frac{1}{2}}$

$u = x^3 - x$

$f'(u) = \frac{1}{2}u^{-1/2} = \frac{1}{2u^{1/2}} = \frac{u^{1/2}}{2u}$

$f'(x) = (3x^2-1)\left(\frac{(x^3-x)^{1/2}}{2(x^3-x)}\right)$

$= \frac{(3x^2-1)(x^3-x)^{1/2}}{2(x^3-x)}$

5. $f(x) = (x^4+2x)^{-3}$

$u = x^4+2x$

$f'(u) = -3u^{-4}$

$f'(x) = (4x^3+2)(-3(x^4+2x)^{-4})$

$= \frac{-12x^3-6}{(x^4+2x)^4}$

BONUS: $f(x) = \ln(\sin(3x+7))$

$\frac{3\cos(3x+7)}{\sin(3x+7)}$