

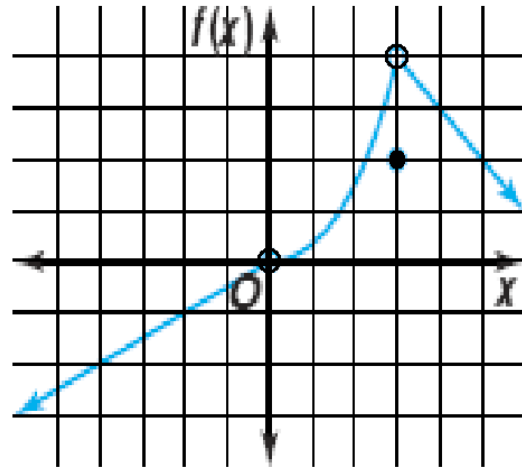
**15.1 - Limits**

1. Use the graph of  $y = f(x)$  to find each pair of values.

a.  $f(-2)$  and  $\lim_{x \rightarrow -2} f(x)$

b.  $f(0)$  and  $\lim_{x \rightarrow 0} f(x)$

c.  $\lim_{x \rightarrow 3} f(x)$  and  $f(3)$



2. Evaluate each limit:

a.  $\lim_{x \rightarrow 2} (-4x^2 - 3x + 6)$

c.  $\lim_{x \rightarrow -1} (-x^3 + 3x^2 - 4)$

b.  $\lim_{x \rightarrow \pi} \frac{\sin x}{x}$

d.  $\lim_{x \rightarrow 0} (x + \cos x)$

3. Evaluate each limit:

a.  $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$

d.  $\lim_{x \rightarrow 0} \frac{x^3 - x^2 + 2x}{x^3 + 4x^2 - 2x}$

b.  $\lim_{x \rightarrow 0} \frac{2n^2}{n}$

e.  $\lim_{x \rightarrow 0} \frac{x \cos x}{x^2 + x}$

c.  $\lim_{x \rightarrow 3} \frac{x^2 - 3x}{x^2 + 2x - 15}$

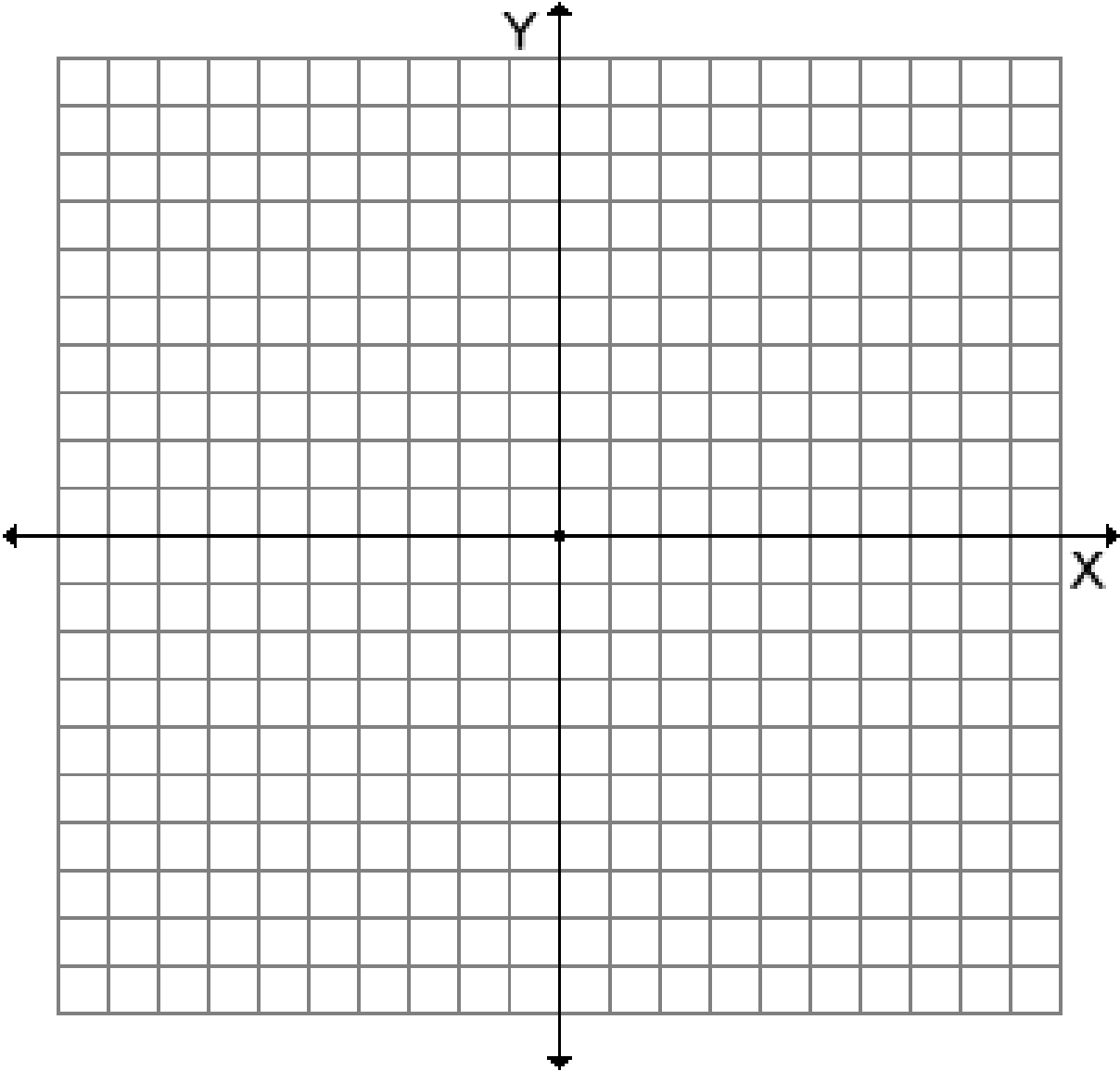
f.  $\lim_{x \rightarrow 0} \frac{(x + 2)^2 - 4}{x}$

**15.2 - Secant Lines**

1. Using  $x^3 - 6x + 2$ , find each Average Rate of Change between each pair of constant points of the function. (You may want to generate an x-y table to make the process go more quickly).

Constant Points	<b>A.R.C. work</b>	Average Rate of Change
-8 to 4		
-6 to 2		
-4 to 0		
-3 to -1		
-2.5 to -1.5		
-2.25 to -1.75		
-2.1 to -1.9		
-2.05 to -1.95		
-2.01 to -1.99		

2. Using the graph paper on the back, carefully graph the function and *lightly* graph the Secant Line associated with each A.R.C.
3. What does your graph tell you about Instantaneous Rate of Change and a possible Tangent Line?



**15.3 - Polynomial Derivative “Tricks”**

Use the Polynomial tricks to find  $f'(x)$  for each  $f(x)$ .

1.  $f(x) = x^7$

2.  $f(x) = 13x^4$

3.  $f(x) = 4$

4.  $f(x) = 3x^2 + 2$

5.  $f(x) = \frac{1}{x^{101}}$

6.  $f(x) = \frac{7}{x^5}$

7.  $f(x) = 5x^{10} + x^5 - 2x^2 + 1$

8.  $f(x) = x^5 - x^4 + x^2 - x + 1$

9.  $f(x) = 12x - 3$

10.  $f(x) = \frac{1}{x^4} - \frac{2}{x^2} + x$

Find the slope of the line tangent to each function at  $x=2$ .

11.  $f(x) = x^7$

12.  $f(x) = x - 2$

13.  $f(x) = x^2 + 2x - 3$

14.  $f(x) = x^{\frac{1}{2}}$

**15.4 - Average Rate of Change (The Basics)**

*For problems 1-3, let the position function of a moving particle be  $S(t) = t^2 - 25$ .  
Find the average velocity from  $t=3$  to*

1.  $t=5$

2.  $t=4$

3.  $t=3.1$

*For problems 4-9, let the position function of a moving particle be  $S(t) = 3t^2 - 2t + 1$ .*

4. Determine a formula for the average velocity (the average rate of change).

5. Use this formula to find the average velocity on the interval  $[0,2]$  which means  $0 \leq x \leq 2$ .

6. Determine a formula for the instantaneous velocity, also known as \_\_\_\_\_.

7. Use this formula to find the instantaneous velocity at  $t=0$ .

8. Determine a formula for the instantaneous acceleration, also know as \_\_\_\_\_.

9. Use this formula to find the instantaneous acceleration at  $t=0$ .

**15.4 (continued) - Velocity and Acceleration**

For each position function, find  $v(t)$  and  $a(t)$ .

	Position Function	Velocity Function	Acceleration Function
1)	$S(t) = 2t^2 + 5t - 12$		
2)	$S(t) = 4t + 3$		
3)	$S(t) = 4 - 2t - t^2$		
4)	$S(t) = (2t + 3)^2$		
5)	$S(t) = (4 - t)^2$		
6)	$S(t) = 64t - 16t^2$		
7)	$S(t) = 2t^3 - 18t^2 + 48t - 6$		
8)	$S(t) = 2t^3 - 9t^2 + 12$		

Use the following information for problems 9 - 11.

If a ball is thrown vertically upward with a velocity of 32 feet per second, the ball's height after  $t$  seconds is given by  $S(t) = 32t - 16t^2$ .

9. What time  $t$  does the ball reach its maximum height?

10. What is the maximum height?

11. At what time does the ball hit the ground?

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**15.4 (continued)**

*Use the following information for problems 12 – 19..*

If a ball is thrown vertically upward with an initial velocity of 128 feet per second, the ball's height after  $t$  seconds is given by  $S(t) = 128t - 16t^2$ .

12. What is the velocity function?
  
13. What is the velocity when  $t =$ 
  - a. 2 seconds
  - b. 4 seconds
  - c. 6 seconds
  
14. At what time is the velocity...
  - a. 48 feet per second?
  - b. 16 feet per second?
  - c. -48 feet per second?
  
15. When is the velocity zero?
  
16. What is the height of the ball when the velocity is zero?
  
17. Is this the maximum height?
  
18. When does the ball hit the ground?
  
19. What is its velocity at that time?

**15.5 - Writing Tangent Lines**

Determine the equation of the line tangent to the given function  $f(x)$  at the point of tangency for the given  $x$  value. Leave your equation in point-slope form!

1.  $f(x) = \frac{1}{2}x^2 + 2x$  at  $x=3$ .

4.  $f(x) = x^3 - x$  at  $x=-1$ .

2.  $f(x) = \frac{1}{2}x^2 + 2x$  at  $x=1$ .

5.  $f(x) = \frac{1}{x^{-2}}$  at  $x=3$ .

3.  $f(x) = x^{\frac{1}{2}}$  at  $x=4$ .



**15.6 - Product Rule**

Function	Derivative
$e^x$	
$\ln x$	
$\sin x$	
$\cos x$	

Use the product rule to find the derivative of the function described by the given equation. Write the answer in a simplified form.

1.  $f(x) = x(x^3 - x^2 + 10)$

4.  $f(x) = (x^2 - x^4)e^x$

2.  $f(x) = (x^2 + x)(x - 3)$

5.  $f(x) = (\ln x)(e^x - 7x)$

3.  $f(x) = (\cos x)(x^2 - 7x)$

**15.7 - Quotient Rule**

Use the quotient rule to find the derivative of each of the function listed below.

1.  $f(x) = \frac{4x + 2}{3x^2}$

3.  $f(x) = \frac{e^x}{2x}$

2.  $f(x) = \frac{x}{x^2 + 2x + 1}$

4.  $f(x) = \frac{x^2 - 2x}{e^x}$

5. Find the slope of the line tangent to the function at the given point:  $f(x) = \frac{\ln x}{x - 1}$  at  $x = 3$

6. Find the coordinates of the points on the graph of  $f(x)$  at which  $f'(x) = 0$ :  $f(x) = \frac{x}{1 + x^2}$

**15.7 (continued) Product AND Quotient Rule**

*Caution: The functions below involve using the product rule along with the quotient rule.*

1.  $f(x) = \frac{xe^x}{x+1}$

2.  $f(x) = \frac{e^x \ln x}{2-x}$

3.  $f(x) = \frac{x-1}{xe^x}$

**15.8 - Chain Rule**

*Use the chain rule to find the derivative of each function.*

1.  $f(x) = (x^2 + 1)^5$

4.  $f(x) = (x^3 - x)^{\frac{1}{2}}$

2.  $f(x) = \cos(4x)$

5.  $f(x) = (x^4 + 2x)^{-3}$

3.  $f(x) = e^{5x}$

**BONUS:**  $f(x) = \ln(\sin(3x + 7))$