Section 11.1: Real Exponents

Objectives:
- I can use the properties of exponents.
- I can evaluate and simplify expressions containing rational (fraction) exponents.
- I can solve equations containing rational (fraction) exponents.

Definition: A number is written in scientific notation when it is in the form $a \times 10^n$, where a is between 1 and 10 and n is an integer.

<u>Example 1:</u>
At their closest points, Mars and Earth are approximately 7.5 x 10^7 km apart.

a. Write this distance in standard form.

b. How many times farther is the distance Mars Pathfinder traveled than the minimum distance between Earth and Mars? (the pathfinder traveled 4.013 x 10^8)

Graphing Calculator Exploration
Graph each of the following sets of equations on the same screen. Use the graphs and tables to determine whether Y1 is equivalent to Y2 or Y3.

1. Y1 = $x^2 x^3$, Y2 = x^5, Y3 = x^6

2. Y1 = $(x^2)^3$, Y2 = x^5, Y3 = x^6

3. Make a conjecture (guess) about the value of $a^m a^n$.

4. Make a conjecture (guess) about the value of $(a^m)^n$.

5. Use your graphing calculator to investigate the value of an expression like $\left(\frac{a}{b}\right)^m$. What do you observe?

<u>Exponent Rules ☺</u>

Property	Definition
Product	
Power of a Power	
Power of a Quotient	
Power of a Product	
Quotient	
Rational Exponent	
Negative Exponent	
Zero Exponent	

<u>Example 2:</u>
Evaluate the expressions:

a. $125^{1/3}$

b. $(s^2t^3)^5$

c. $\left(\frac{2}{5}\right)^{-1}$

d. $\left(\frac{x^3y}{(x^4)^3}\right)$

e. $(81c^4)^{1/4}$

f. $625^{3/4}$

g. Simplify: $\sqrt{r^7s^{25}t^3}$

h. Solve: $734 = x^{3/4} + 5$

Section 11.2 &4: Exponential and Logarithmic Functions (Properties)

Objectives:
- I can solve problems involving exponential growth and decay.
- I can evaluate expressions involving logarithms.
- I can solve equations and inequalities involving logarithms.

Definition: An **exponential function** is of the form $y = b^x$ where the base b is a positive real number and the exponent is a variable.

Graphing Calculator Exploration

Directions: Graph $y = b^x$ for $b = 0.5, 0.75, 2$, and 5 on the same screen. Then answer the questions below:

1. What is the range of each exponential function?

2. What point is on the graph of each function? (What point do they share in common?)

3. What is the end behavior of each graph?

4. Do the graphs have any asymptotes (horizontal or vertical)?

5. Is the range of every exponential function the same? Explain.

6. Why is the point at (0, 1) on the graph of every exponential function?

7. For what values of a is the graph of $y = a^x$ increasing and for what values is the graph decreasing? Explain.

8. Explain the existence or absence of the asymptotes in the graph of an exponential function.

<u>Characteristics of an Exponential Function</u>:

	b > 1	0 < b < 1
Domain		
Range		
y-intercept		
Behavior		
Horizontal Asymptote		
Vertical Asymptote		

What happens when $b = 1$?

<u>Anatomy of a logarithm:</u>

$$y = \log_b x \longrightarrow b^y = x$$

<u>Example 1:</u>

Write each expression in exponential form:

a. $\log_{125} 25 = \frac{2}{3}$

b. $\log_8 2 = \frac{1}{3}$

<u>Example 2:</u>

Write each equation in logarithmic form:

a. $4^3 = 64$

b. $3^{-3} = \frac{1}{27}$

<u>Example 3:</u>

Evaluate the expression $\log_7 \frac{1}{49} = y$.

Example 4:
How long would it take for 256,000 grams of Thorium-234, with a half-life of 25 days, to decay to 1000 grams?

Properties of Logarithms

Property	Definition
Product	
Quotient	
Power	
Equality	

Example 5:

Solve each equation:

a. $\log_p 64^{\frac{1}{3}} = \frac{1}{2}$

b. $\log_4(2x+11) = \log_4(5x-4)$

c. $\log_{11} x + \log_{11}(x+1) = \log_{11} 6$

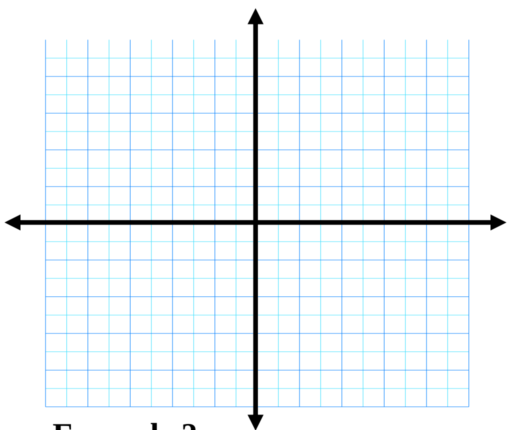
Section 11.2 & 4: Exponential and Logarithmic Functions (Graphing)

Objectives:

- Graph exponential functions and inequalities.
- Graph logarithmic functions and inequalities.

Example 1:

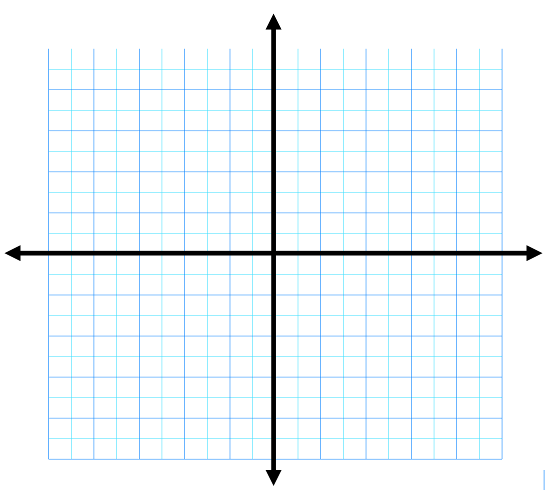
Graph the exponential functions $y = 4^x$, $y = 4^x + 2$, and $y = 4^x - 3$ on the same set of axes.



Compare and Contrast the graphs:

Example 2:

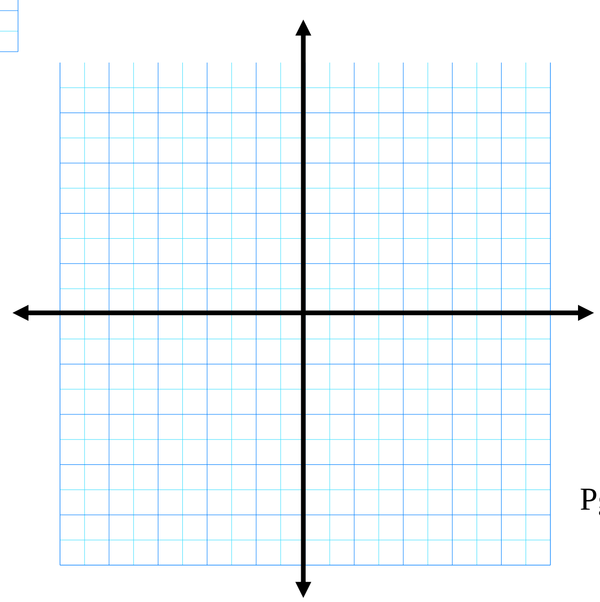
Graph the exponential functions $y = \left(\frac{1}{5}\right)^x$, $y = 6\left(\frac{1}{5}\right)^x$, & $y = -2\left(\frac{1}{5}\right)^x$ on the same axes.



Compare and contrast the graphs.

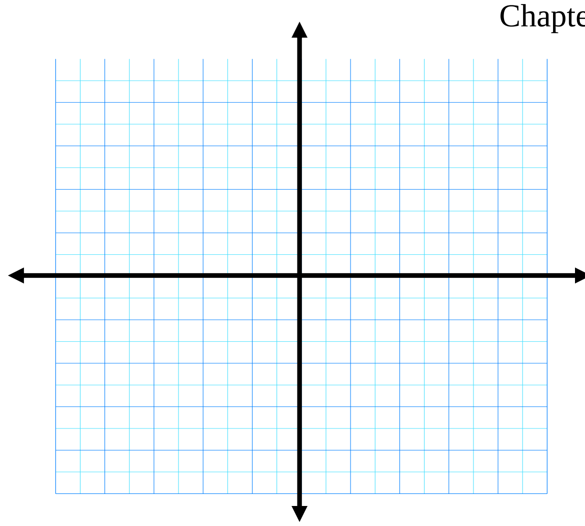
Example 3:

Graph $y < 2^x + 1$.



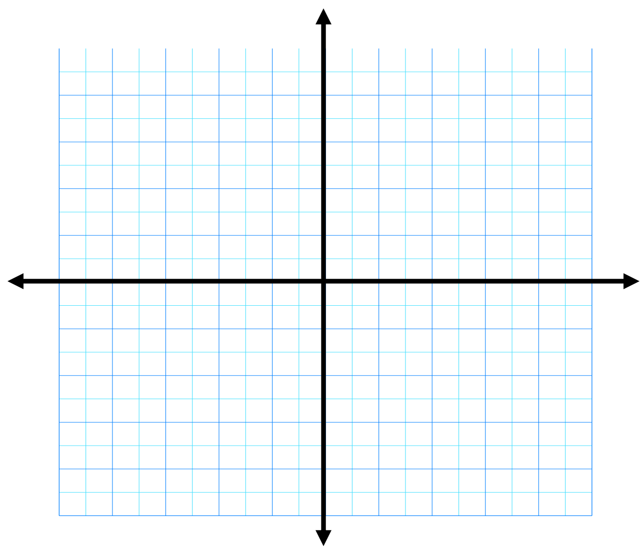
Example 4:

Graph $y = 3^x$



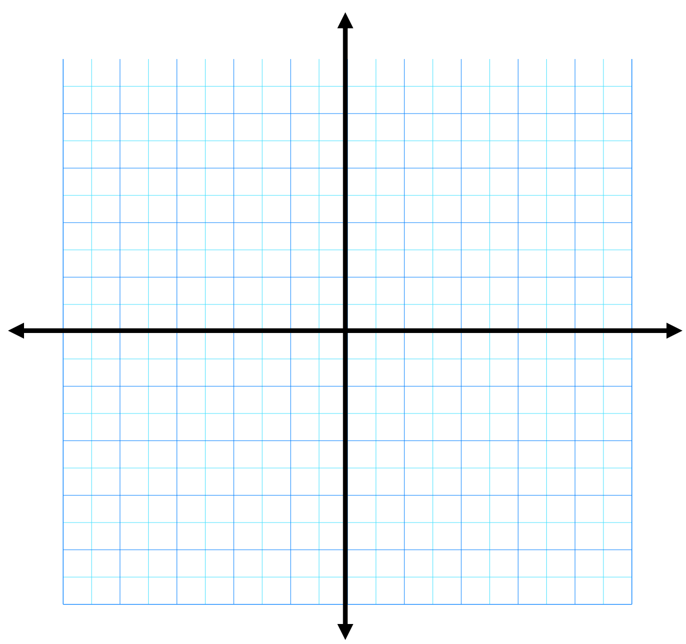
Example 5:

Graph $y = \log_3(x+1)$.



Example 6:

Graph $y \le \log_5 x - 2$

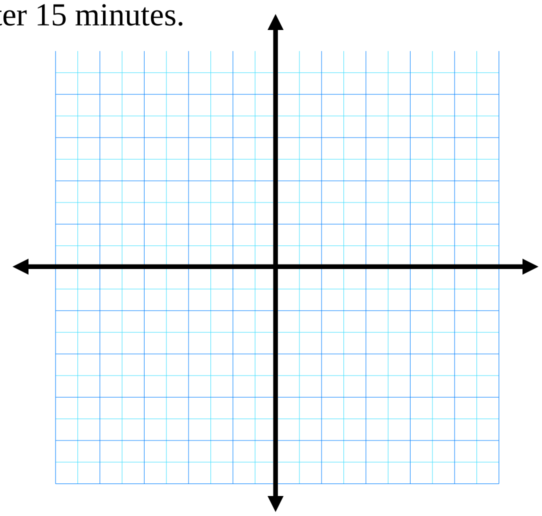


Section 11.2 & 4: Exponential and Logarithmic Functions (Application)

Example 1:
According to Newton's Law of Cooling, the difference between the temperature of an object and its surroundings decreases in time exponentially. Suppose a certain cup of coffee is 95ºC and it is in a room that is 20ºC. The cooling for this particular cup can be modeled by the equation $y = 75(0.875)^t$ where y is the temperature difference and t is the time in minutes.

a. Find the temperature of the coffee after 15 minutes.

b. Graph the cooling function.



Definition: The equation $A = P\left(1+\frac{r}{n}\right)^{nt}$, where A is the final amount, P is the initial (principal) amount, r is the annual interest rate, n is the number of times interest is paid/compounded per year, and t is the number of time periods, is used for modeling compound interest.

Example 2:
Suppose that a researcher estimates that the initial population of varroa in a colony is 500. They are increasing at a rate of 14% per week. What is the expected population in 22 weeks?

Example 3:
Determine the amount of money in a money market account providing an annual rate of 5% compounded daily if Marcus invested $2000 and left it in the account for 7 years.

Section 11.5: Common Logarithms

Objectives:

- I can find common logarithms of numbers.
- I can solve equations and inequalities using common logarithms.
- I can solve real-world applications with common logarithmic functions.

Definition: A **common logarithm** is a logarithm with a base of 10.

<u>Example 1:</u>
Given that $\log 7 = 0.8451$, evaluate each logarithm.

a. log 7,000,000

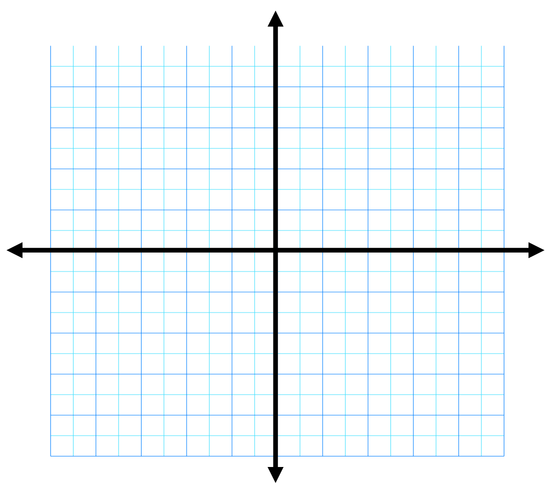
b. log 0.0007

<u>Example 2:</u>
Evaluate each expression.

a. $\log 5(2)^3$

b. $\log \dfrac{19^2}{6}$

<u>Example 3:</u>
Graph $y > \log(x - 4)$.



Definition: If a, b, and n are positive numbers and neither a nor b is 1, then the following equation is true: $\log_a n = \dfrac{\log_b n}{\log_b a}$.

Example 4:
Find the value of $\log_9 1043$ using the change of base formula.

Example 5:
Solve the following equations:
a. $6^{3x} = 81$

b. $5^{4x} = 73$

c. $8^{4x} = 3$

Section 11.3 & 6: The Number *e* and Natural Logarithms

Objectives:

- I can use the exponential function $y = e^x$.
- I can find natural logarithms of numbers.
- I can solve equations and inequalities using natural logarithms.
- I can solve real-world applications with natural logarithms.

Definition: The formula for **exponential growth and decay** (in terms of *e*) is $A = Pe^{rt}$ where A is the final amount, P is the initial (principal) amount, r is the rate constant, and t is time. (HINT: Look for the phrase "compounded continuously" when using this formula.)

On your calculator, find the *e* button. What number does the calculator give you? ______.

The number *e* is JUST A NUMBER… just like π is JUST A NUMBER.

<u>Example 1:</u>
Compare the balance after 25 years of a $10,000 investment earning 6.75% interest compounded continuously to the same investment compounded semiannually.

<u>Example 2:</u>
According to Newton, a beaker of liquid cools exponentially when removed from a source of heat. Assume that the initial temperature is 90°F and that $r = 0.275$.

a. Write a function to model the rate at which the liquid cools.

b. Find the temperature of the liquid after 4 minutes.

Definition: A **natural logarithm** is a logarithm with a base of *e* and is denoted ln *x*.

Example 1:

Convert $\log_6 254$ to a natural logarithm and evaluate.

Example 2:

Solve: $6.5 = -16.25 \ln x$

Example 3:

Solve each equation or inequality by using natural logarithms.

a. $3^{2x} = 7^{x-1}$

b. $6^{x^2-2} < 48$