Section 11.1 (Rational Exponents) **Name** Key

Evaluate each expression.

1. $\dfrac{8^{\frac{2}{3}}}{8^{\frac{1}{3}}}$

$8^{1/3}$

$\sqrt[3]{8}$

$\boxed{2}$

2. $\left(\dfrac{4}{5}\right)^{-2}$

$\dfrac{5^2}{4^2}$

$\boxed{\dfrac{25}{16}}$

3. $343^{\frac{2}{3}}$

$\left(\sqrt[3]{343}\right)^2$

$(7)^2$

$\boxed{49}$

4. $\sqrt[3]{8^3}$

$\boxed{8}$

5. $\sqrt{5} \cdot \sqrt{10}$

$\sqrt{50}$

$\sqrt{2 \cdot 25}$

$\boxed{5\sqrt{2}}$

6. $9^{-\frac{1}{2}}$

$\dfrac{1}{\sqrt{9}}$

$\boxed{\dfrac{1}{3}}$

Simplify each expression.

7. $(5n^3)^2 \cdot n^{-6}$

$25n^6 \cdot n^{-6}$

$25n^0$

$\boxed{25}$

8. $\left(\dfrac{x^2}{4y^{-2}}\right)^{-\frac{1}{2}}$

$\left(\dfrac{x^2y^2}{4}\right)^{-\frac{1}{2}}$

$\sqrt{\dfrac{4}{x^2y^2}}$

$\boxed{\dfrac{2}{xy}}$

9. $(64x^6)^{\frac{1}{3}}$

$64^{1/3}x^2$

$\sqrt[3]{64} \cdot x^2$

$\boxed{4x^2}$

10. $(5x^6y^4)^{\frac{1}{2}}$

$\sqrt{5x^6y^4}$

$\boxed{|x^3|y^2\sqrt{5}}$

11. $\sqrt{x^2y^3} \cdot \sqrt{x^3y^4}$

$\sqrt{x^5y^7}$

$\boxed{x^2|y^3|\sqrt{xy}}$

12. $\left(\dfrac{p^{6a}}{p^{-3a}}\right)^{\frac{1}{3}}$

$(p^{9a})^{1/3}$

$p^{\frac{9a}{3}}$

$\boxed{p^{3a}}$

Express each using rational exponents.

13. $\sqrt{x^5y^6}$

$(x^5y^6)^{1/2}$

$\boxed{x^{5/2}y^3}$

14. $\sqrt[5]{27x^{10}y^5}$

$\boxed{27^{1/5}x^2y}$

15. $\sqrt{144x^6y^{10}}$

$\boxed{12x^3y^5}$

16. $21\sqrt[3]{c^7}$

$\boxed{21c^2c^{1/3}}$

or

$\boxed{21c^{7/3}}$

17. $\sqrt{1024a^3}$

$\boxed{32a\sqrt{a}}$

18. $\sqrt[4]{36a^8b^5}$

$\boxed{36^{1/4}a^2b^{5/4}}$

Express each using radicals.

19. $64^{\frac{1}{3}}$

$\boxed{\sqrt[3]{64}}$

20. $2^{\frac{1}{2}}a^{\frac{3}{2}}b^{\frac{5}{2}}$

$\boxed{\sqrt{2a^3b^5}}$

21. $s^{\frac{2}{3}}t^{\frac{1}{3}}v^{\frac{2}{3}}$

$\boxed{\sqrt[3]{s^2tv^2}}$

22. $y^{\frac{3}{2}}$

$\boxed{\sqrt[2]{y^3}}$

or

$\boxed{(\sqrt{y})^3}$

23. $x^{\frac{2}{5}}y^{\frac{3}{5}}$

$\boxed{\sqrt[5]{x^2y^3}}$

24. $(x^6y^3)^{\frac{1}{2}}z^{\frac{3}{2}}$

$x^3y^{3/2}z^{3/2}$

$\boxed{x^3\sqrt{y^3z^3}}$

Section 11.2 and 11.4 (Rules and Properties – Day 1) Name Key

Logarithmic Functions

Write each equation in exponential form.

1. $\log_3 81 = 4$

$3^4 = 81$

2. $\log_8 2 = \frac{1}{3}$

$8^{1/3} = 2$

3. $\log_{10} \frac{1}{100} = -2$

$10^{-2} = \frac{1}{100}$

Write each equation in logarithmic form.

4. $3^3 = 27$

$\log_3 27 = 3$

5. $5^{-3} = \frac{1}{125}$

$\log_5 \frac{1}{125} = -3$

6. $\left(\frac{1}{4}\right)^{-4} = 256$

$\log_{\frac{1}{4}} 256 = -4$

Evaluate each expression.

7. $\log_7 7^3 = x$

$7^x = 7^3$

$\boxed{x = 3}$

8. $\log_{10} 0.001 = x$

$10^x = .001$

$10^x = 10^{-3}$

$\boxed{x = -3}$

9. $\log_8 4096 = x$

$8^x = 4096$

$8^x = 8^4$

$\boxed{x = 4}$

10. $\log_4 32 = x$

$4^x = 32$

$(2^2)^x = 2^5$

$2x = 5$

$\boxed{x = 2.5}$

11. $\log_3 1 = x$

$3^x = 1$

$\boxed{x = 0}$

12. $\log_6 \frac{1}{216} = x$

$6^x = \frac{1}{216}$

$6^x = 6^{-3}$

$\boxed{x = -3}$

Solve each equation.

13. $\log_x 64 = 3$

$x^3 = 64$

$\boxed{x=4}$

14. $\log_4 0.25 = x$

$4^x = \frac{1}{4}$

$4^x = 4^{-1}$

$\boxed{x=-1}$

15. $\log_4(2x-1) = \log_4 16$

$2x-1 = 16$

$2x = 17$

$\boxed{x=8.5}$

16. $\log_{10}\sqrt{10} = x$

$10^x = \sqrt{10}$

$10^x = 10^{1/2}$

$\boxed{x=\frac{1}{2}}$

17. $\log_7 56 - \log_7 x = \log_7 4$

$\log_7 \frac{56}{x} = \log_7 4$

$\frac{56}{x} = 4$

$\frac{56}{4} = x$

$\boxed{x=14}$

18. $\log_5(x+4) + \log_5 x = \log_5 12$

$\log_5(x^2+4x) = \log_5 12$

$x^2+4x = 12$

$x^2+4x-12 = 0$

$(x-2)(x+6) = 0$

$x-2=0 \qquad x+6=0$

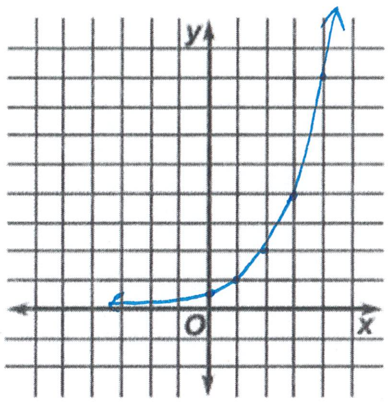
$\boxed{x=2} \qquad \cancel{x=-6}$

Reject

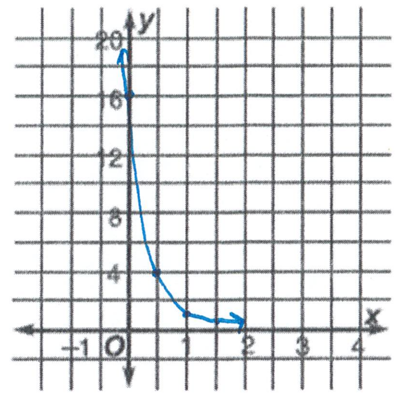
Section 11.2 and 11.4 (Graphing – Day 2) Name Key

Graph each exponential function or inequality.

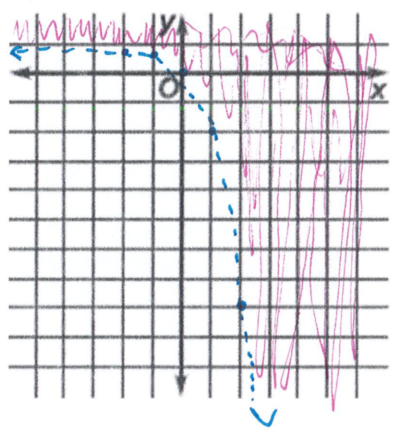
1. $y = 2^{x-1}$



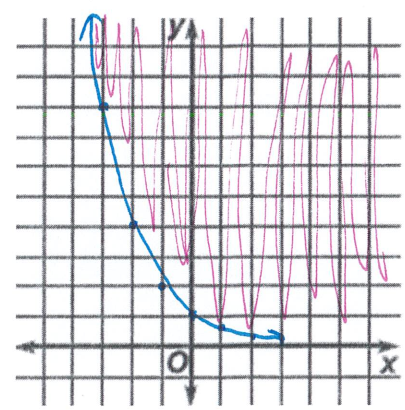
2. $y = 4^{-x+2}$



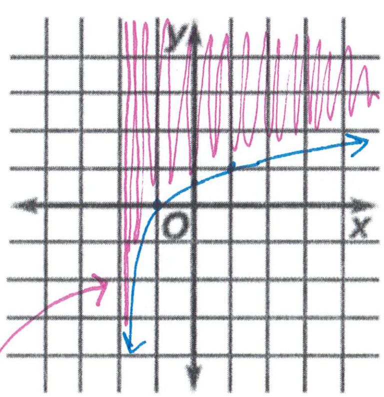
3. $y > -3^x + 1$



4. $y \geq 0.5^x$



5. $y \geq \log_3(x+2)$



Don't shade left of the asymptote.

Section 11.2 and 11.4 (Applications – Day 3) **Name** Key

1. ***Demographics*** An area in North Carolina known as The Triangle is principally composed of the cities of Durham, Raleigh, and Chapel Hill. The Triangle had a population of 700,000 in 1990. The average yearly rate of growth is 5.9%. Find the projected population for 2010.

$y = 700000(1+.059)^{20}$

$\boxed{2,203,014 \text{ people}}$

2. ***Finance*** Determine the amount of money in a savings account that provides an annual rate of 4% compounded monthly if the initial investment is \$1000 and the money is left in the account for 5 years.

$A = 1000\left(1+\frac{.04}{12}\right)^{12 \cdot 5}$

$A = 1220,996$

$\boxed{A \approx \$1221.00}$

3. ***Investments*** How much money must be invested by Mr. Kaufman if he wants to have \$20,000 in his account after 15 years? He can earn 5% compounded quarterly.

$20000 = P\left(1+\frac{.05}{4}\right)^{4 \cdot 15}$

$\frac{20000}{2.107181347} = \frac{2.107181347P}{2.107181347}$

$\boxed{P = \$9491.35}$

4. ***Chemistry*** How long would it take 100,000 grams of radioactive iodine, which has a half-life of 60 days, to decay to 25,000 grams? Use the formula $N = N_0\left(\frac{1}{2}\right)^t$, where N is the final amount of the substance, N_0 is the initial amount, and *t* represents the number of half-lives.

$\frac{25000}{100000} = \frac{100000\left(\frac{1}{2}\right)^t}{100000}$

$\frac{1}{4} = \frac{1}{2}^t$

$\left(\frac{1}{2}\right)^2 = \frac{1}{2}^t$

$t = 2$

$2(60) = \boxed{120 \text{ days}}$

Section 11.5 (Common Logs) Name Key

Given that log 3 = 0.4771, log 5 = 0.6990, and log 9 = 0.9542, evaluate each log.

1. $\log 300{,}000 = x$

$\log 3 + \log 100000 = x$

$\log 3 + \log 10^6 = x$

$.4771 + 6 = x$

$\boxed{x = 6.4771}$

2. $\log 9000 = \log 9 + \log 1000$

$= \log 9 + \log 10^3$

$= .9542 + 3$

$= \boxed{3.9542}$

3. $\log 75 = \log 25 + \log 3$

$= \log 5^2 + \log 3$

$= 2 \cdot \log 5 + \log 3$

$= 2(.6990) + (.4771)$

$= \boxed{1.8751}$

4. $\log 0.0005$

$\log 5 + \log 10^{-4}$

$.6990 + -4$

$\boxed{-3.301}$

5. $\log 27$

$\log 3^3$

$3 \log 3$

$3(.4771)$

$\boxed{1.4313}$

6. $\log 81$

$\log 9^2$

$2 \log 9$

$2(.9542)$

$\boxed{1.9084}$

Evaluate each expression.

7. $\log 66.3 = x$

$10^x = 66.3$

$x = \dfrac{\log 66.3}{\log 10}$

$\boxed{x = 1.82}$

8. $\log \leq \dfrac{17^4}{5}$

SKIP

9. $\log 7(4^3) = x$

$10^x = 448$

$x = \dfrac{\log 448}{\log 10}$

$\boxed{x \approx 1.68}$

Find the value of each logarithm using the change of base formula.

10. $\log_6 832 = x$

$6^x = 832$

$x = \dfrac{\log 832}{\log 6}$

$\boxed{x \approx 3.7526}$

11. $\log_{11} 47 = x$

$11^x = 47$

$x = \dfrac{\log 47}{\log 11}$

$\boxed{x \approx 1.6056}$

12. $\log_3 9 = x$

$3^x = 9$

$x = \dfrac{\log 9}{\log 3}$

$\boxed{x = 2}$

Solve each equation or inequality.

13. $8^x = 10$

$$x = \frac{\log 10}{\log 8}$$

$$\boxed{x \approx 1.1073}$$

14. $1.8^{x-5} = 19.8$

$$x - 5 = \frac{\log 19.8}{\log 1.8}$$

$$x - 5 = 5.0795$$

$$\boxed{x = 10.0795}$$

15. $4^{2x} > 25$

$$2x \cdot \log 4 > \log 25$$

$$2x > \frac{\log 25}{\log 4}$$

$$\frac{2x}{2} > \frac{2.32}{2}$$

$$\boxed{x > 1.16}$$

16. $2.4^x \leq 20$

$$x \cdot \log 2.4 \leq \log 20$$

$$\boxed{x \leq 3.42}$$

17. $3^{5x} = 85$

$$5x = \frac{\log 85}{\log 3}$$

$$\frac{5x}{5} = \frac{4.0439}{5}$$

$$\boxed{x = .8088}$$

18. $3^{2x-2} = 2\overset{x}{\cancel{x}}$

$$(2x-2) \cdot \log 3 = x \cdot \log 2$$

$$2x - 2 = x(.6309)$$

$$1.3691x = 2$$

$$\boxed{x = 1.4608}$$

Section 11.3 and 11.6 (Natural Logs) **Name** Key

1. In 1995, the population of Kalamazoo, MI, was 79, 089. This figure represented a 0.4% annual decline from 1990.

   a. Let *t* be the number of years since 1995 and write a function that models the population in Kalamazoo in 1995.

   $P = 79089(1-.004)^t$

   b. Predict the population in 2010 and in 2015. Assume a steady rate of decline.

   $P = 79089(.996)^{15}$

   $\boxed{P \approx 74,474}$

   $P = 79089(.996)^{20}$

   $\boxed{P \approx 72,997}$

2. Suppose Karen deposits $1500 in a savings account that earns 6.75% interest compounded continuously. She plans to withdraw the money in 6 years to make a $2500 down payment on a car. Will there be enough funds for Karen to meet her goal?

   $A = 1500e^{(.0675 \cdot 6)}$

   $A \approx \$2248.95$ $\boxed{\text{No}}$

3. Given the original principal, the annual interest rate, the amount of time for each investment, and the type of compounded interest, find the amount of each account.

   a. P = $1250, r = 8.5%, t = 3 years, semiannually.

   $A = 1250\left(1 + \frac{.085}{2}\right)^{2 \cdot 3}$

   $\boxed{A \approx \$1604.50}$

   b. P = $2575, r = 6.25%, t = 5 years 3 months, continuously.

   $A = 2575e^{.0625 \cdot 5.25}$

   $\boxed{A \approx \$3575.03}$

Evaluate each expression.

4. ln 71 = x

   $e^x = 71$

   $x = \frac{\ln 71}{\ln e}$

   $\boxed{x \approx 4.2627}$

5. ln 8.76 = x

   $e^x = 8.76$

   $x = \frac{\ln 8.76}{\ln e}$

   $\boxed{x \approx 2.1691}$

6. ln 0.532 = x

   $e^x = .532$

   $x = \frac{\ln .532}{\ln e}$

   $\boxed{x \approx -.6311}$

Convert each log to a natural log.

7. $\log_4 94 = x$

$4^x = 94$

$\boxed{x = \frac{\ln 94}{\ln 4}}$

8. $\log_5 256 = x$

$5^x = 256$

$\boxed{x = \frac{\ln 256}{\ln 5}}$

9. $\log_9 0.712 = x$

$9^x = .712$

$\boxed{x = \frac{\ln .712}{\ln 9}}$

Use natural logs to solve each equation.

10. $6^x = 42$

$x = \frac{\ln 42}{\ln 6}$

$\boxed{x = 2.086}$

11. $7^x = 4^{x+3}$

$x \cdot \ln 7 = (x+3) \ln 4$

$x = (x+3)(.7124)$

$x = .7124x + 2.1372$

$.2876x = 2.1372$

$\boxed{x \approx 7.4312}$

12. $1249 = 175e^{-0.04t}$

$7.1371 = e^{-.04t}$

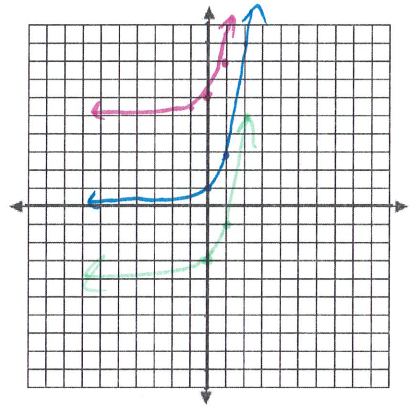
$\ln 7.1371 = -.04t \cdot \ln e$

$1.9653 = -.04t$

$\boxed{t \approx -49.1}$

Exponential Extras

1. Graph the exponential functions $\boxed{y = 3^x}$, $\boxed{y = 3^x + 5}$, and $\boxed{y = 3^x - 4}$ on the same set of axes. In complete sentences, compare and contrast the graphs. *LABEL EACH*



2. Between 1990 and 2000, the population of Florida had an annual growth rate of about 2.14%. If the state's population was 15,989,069 in 2000, approximately what was Florida's population in 1990?

$$15989069 = a(1.0214)^{10}$$

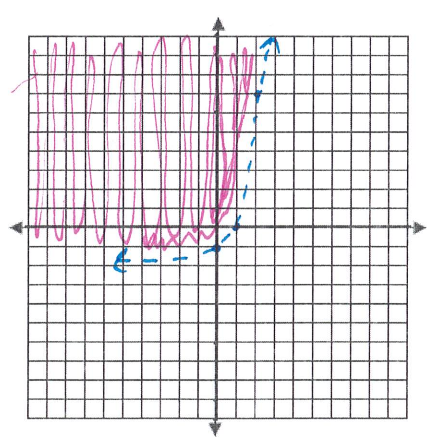
$$\boxed{a \approx 12{,}937{,}925}$$

3. Determine the amount of money in a savings account providing an annual rate of 3% compounded daily if Sandra made a one-time deposit of $8500 in to the account and left it there for 5 years.

$$A = 8500\left(1 + \frac{.03}{365}\right)^{365 \cdot 5}$$

$$\boxed{A \approx \$9875.53}$$

4. Graph $y > 3^x - 2$.



5. Jared purchases a new car for \$26,400. The car <u>loses</u> 21.5% of its value each year.

a. Write a function to model the VALUE of the car.

$y = 26400(1-.215)^x$

b. Find the value of the car after 4 months of ownership

$y = 26400(.785)^{1/3}$

$\boxed{y = \$24353.45}$

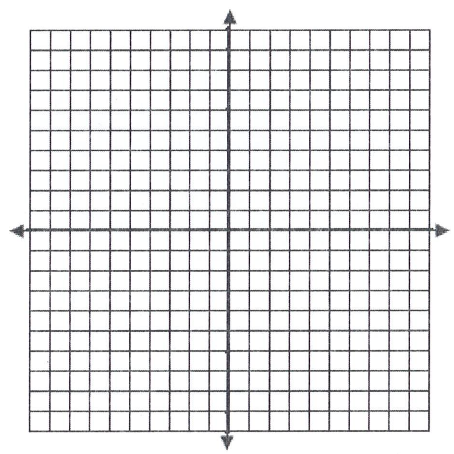
c. Find the value of the car after three years of ownership.

$y = 26400(.785)^3$

$\boxed{y = \$12770.65}$

d. Graph the function and use the graph to verify your answer in part b.

On Desmos



6. Compare the balance after 15 years of a \$20,000 investment earning 7% interest compounded continuously to the same investment compounded quarterly.

$A = 20000\left(1+\frac{.07}{4}\right)^{4 \cdot 15}$

$A \approx \$56636.33$

$A = 20000e^{.07 \cdot 15}$

$A \approx \$57153.02$

Continuously will get you \$516.69 more

7. Write each equation in exponential form.

a. $\log_{32} 8 = \frac{3}{5}$

$32^{3/5} = 8$

b. $\log_{81} 3 = \frac{1}{4}$

$81^{1/4} = 3$

8. Write each equation in logarithmic form.

a. $6^4 = 1296$

$\log_6 1296 = 4$

b. $2^{-8} = \frac{1}{256}$

$\log_2 \frac{1}{256} = -8$

9. Evaluate the expression $\log_3 \frac{1}{27}$. $= x$

$3^x = \frac{1}{27}$

$3^x = 3^{-3}$

$\boxed{x = -3}$

10. Given that log 4 = 0.6021, evaluate the logarithm: log 40,000

$\log 4 + \log 10000$

$\log 4 + \log 10^4$

$.6021 + 4$

$\boxed{4.6021}$

11. Evaluate each expression.

a. $\log 4(3)^4 = x$

$10^x = 324$

$x \cdot \log 10 = \log 324$

$x = \frac{\log 324}{\log 10}$

$\boxed{x = 2.5105}$

b. $\log \frac{22^3}{4} = x$

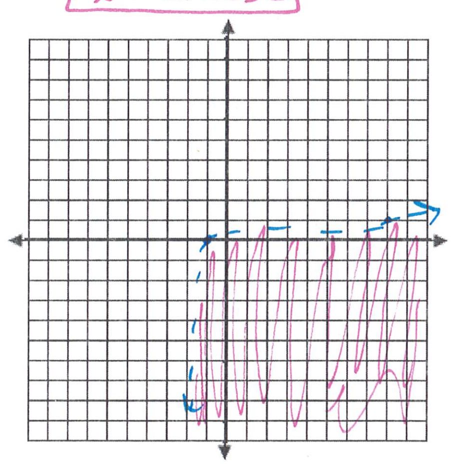
$10^x = 2662$

$x \cdot \log 10 = \log 2662$

$x = \frac{\log 2662}{\log 10}$

$\boxed{x = 3.4252}$

12. Graph $y < \log(x + 2)$.



13. Find the value of $\log_8 550$ using the change of base formula.

$$\frac{\log 550}{\log 8} \approx \boxed{3.0344}$$

14. Convert $\log_4 325$ to a natural logarithm and evaluate.

$$\frac{\ln 325}{\ln 4} \approx \boxed{4.1721}$$

15. Solve $6.7 = -8.2 \ln x$.

$$-.81707 = \ln x$$

$$e^{-.81707} = x$$

$$\boxed{x \approx .4417}$$

16. Solve the equation by using natural logarithms: $4^{3x} = 5^{x-8}$

$$3x \cdot \ln 4 = (x-8) \ln 5$$

$$3x = (x-8)(1.16096)$$

$$3x = 1.16096x - 9.28768$$

$$1.83904x = -9.28768$$

$$\boxed{x \approx -5.0503}$$