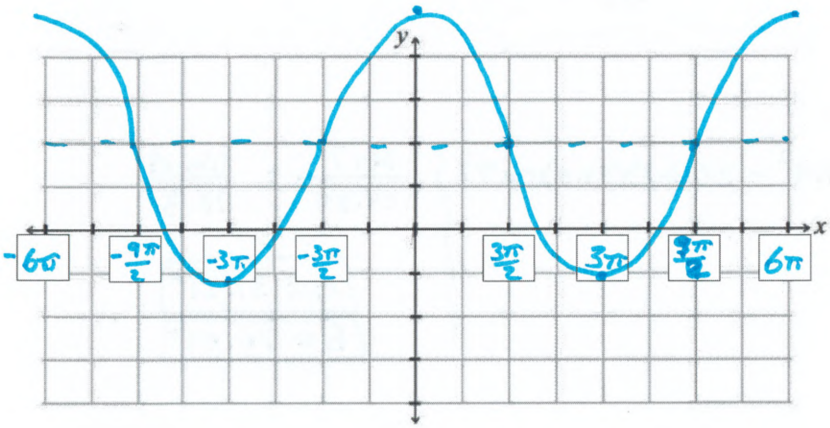
Advanced Math 2nd Semester
Exam Review

Chapter 6: Graphs of the Trigonometric Functions

$y=3\cos\left(\frac{1}{3}\theta\right)+2$ $\frac{2\pi}{\frac{1}{3}} = 6\pi$

6) State the amplitude, period, phase shift and vertical shift of $y = 3\cos(\frac{\theta}{3}) + 2$. Then graph it.



Find the amplitude. 3

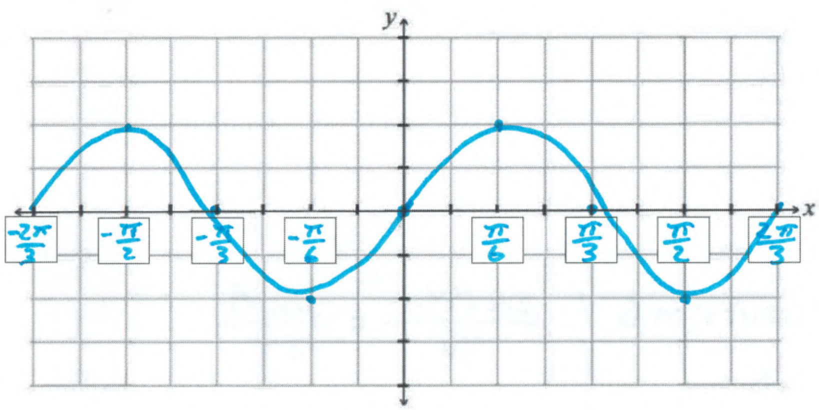
Find the period. 6π

Find the phase shift. none

Find the vertical translation. 2

$y=-2\sin\left(3\left(\theta+\frac{\pi}{3}\right)\right)$ $\frac{2\pi}{3}$

7) State the amplitude, period, phase shift and vertical shift of $y = -2\sin(3\theta + \pi)$. Then graph it.



Find the amplitude. 2 (flip)

Find the period. $\frac{2\pi}{3}$

Find the phase shift. $-\frac{\pi}{3}$

Find the vertical translation. none

8) Write an equation for the sine function with an amp 2.4, period 8.2, phase shift $\frac{\pi}{3}$, and a vertical shift 0.2.

$\frac{2\pi}{8.2} = \frac{\pi}{4.1}$

$y = 2.4\sin\left(\frac{\pi}{4.1}\left(\theta - \frac{\pi}{3}\right)\right) + .2$

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Chapter 7: Trigonometric Identities and Equations

9) No Calculator: Solve $2\sin^2 x - \cos x - 1 = 0$, over the set of real numbers.

$2(1-\cos^2 x) - \cos x - 1 = 0$

$2 - 2\cos^2 x - \cos x - 1 = 0$

$-2\cos^2 x - \cos x + 1 = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0 \qquad \cos x + 1 = 0$

$\cos x = \frac{1}{2} \qquad \cos x = -1$

$\boxed{\begin{array}{c} \pm\frac{\pi}{3} \pm 2\pi n \\ \pi \pm 2\pi n \end{array}}$ or $\boxed{\begin{array}{c} \pm 60 \pm 360n \\ 180 \pm 360n \end{array}}$

10) No Calculator: Simplify the expression and state the domain: $\sin x \sec x =$

$= \sin x \cdot \frac{1}{\cos x}$

$= \frac{\sin x}{\cos x}$

$= \boxed{\tan x}$

$\cos x \neq 0$

$\boxed{D: x \neq \frac{\pi}{2} + \pi n}$

11) No Calculator: Solve over the set of real numbers $(\sin x + 1)(\tan x - 1) = 0$

$\sin x + 1 = 0 \qquad \tan x - 1 = 0$

$\sin x = -1 \qquad \tan x = 1$

$x = \frac{3\pi}{2} \qquad x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\boxed{\begin{array}{c} \frac{3\pi}{2} + 2\pi n \\ \frac{\pi}{4} + \pi n \end{array}}$

12) No Calculator: Solve over the set of real numbers $\cos 2x + 7\sin x - 4 = 0$

$1 - 2\sin^2 x + 7\sin x - 4 = 0$

$-2\sin^2 x + 7\sin x - 3 = 0$

$2\sin^2 x - 7\sin x + 3 = 0$

$(2\sin x - 1)(\sin x - 3) = 0$

$2\sin x - 1 = 0 \qquad \sin x - 3 = 0$

$2\sin x = 1 \qquad \cancel{\sin x = 3}$

$\sin x = \frac{1}{2}$

$\boxed{\begin{array}{c} \frac{\pi}{6} + 2\pi n \\ \frac{5\pi}{6} + 2\pi n \end{array}}$

13) No Calculator: Solve over the set of real numbers $\cos x = 0$

$\boxed{\frac{\pi}{2} + \pi n}$

14) No Calculator: Solve over the set of real numbers $\sin 2x = \frac{1}{2}$

$2x = \frac{\pi}{6} \qquad 2x = \frac{5\pi}{6}$

$x = \frac{\pi}{12} \qquad x = \frac{5\pi}{12}$

$\boxed{\begin{array}{c} \frac{\pi}{12} + 2\pi n \\ \frac{5\pi}{12} + 2\pi n \end{array}}$

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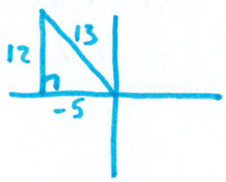
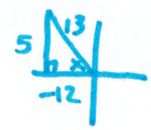
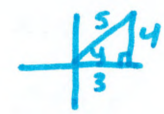
15) Given $\frac{\pi}{2} \le x \le \pi$ and $\cos x = -\frac{5}{13}$, find $\csc x$.



$$\boxed{\csc x = \frac{13}{12}}$$

16) Suppose $\frac{\pi}{2} \le x \le \pi$ and $\sin x = \frac{5}{13}$. Also suppose $0 \le y \le \frac{\pi}{2}$ and $\cos y = \frac{3}{5}$.





a. Find $\sin(x - y)$.

$$\left(\frac{5}{13}\right)\left(\frac{3}{5}\right) - \left(\frac{-12}{13}\right)\left(\frac{4}{5}\right)$$

$$\frac{15}{65} + \frac{48}{65}$$

$$\boxed{\frac{63}{65}}$$

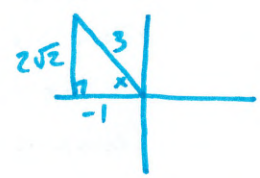
b. Find $\cos(x - y)$.

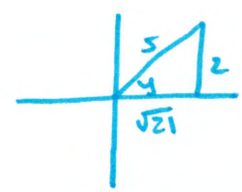
$$\left(\frac{-12}{13}\right)\left(\frac{3}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$$

$$\frac{-36}{65} + \frac{20}{65}$$

$$\boxed{\frac{-16}{65}}$$

17) Suppose x and y are on the intervals $\frac{\pi}{2} < x < \pi$ and with $0 < y < \frac{\pi}{2}$, with $\cos x = -\frac{1}{3}$ and $\sin y = \frac{2}{5}$.





a. Find cos(x + y)

$$\left(-\frac{1}{3}\right)\left(\frac{\sqrt{21}}{5}\right) - \left(\frac{2\sqrt{2}}{3}\right)\left(\frac{2}{5}\right)$$

$$\frac{-\sqrt{21}}{15} - \frac{4\sqrt{2}}{15} = \boxed{\frac{-\sqrt{21} - 4\sqrt{2}}{15}}$$

b. Find sin (x – y)

$$\left(\frac{2\sqrt{2}}{3}\right)\left(\frac{\sqrt{21}}{5}\right) - \left(-\frac{1}{3}\right)\left(\frac{2}{5}\right)$$

$$\frac{2\sqrt{42}}{15} + \frac{2}{15} = \boxed{\frac{2\sqrt{42}+2}{15} \text{ or } \frac{2(\sqrt{42}+1)}{15}}$$

c. Find sin (2x)

$$2\left(\frac{2\sqrt{2}}{3}\right)\left(-\frac{1}{3}\right) = \boxed{\frac{-4\sqrt{2}}{9}}$$

d. Find cos (2y)

$$1 - 2\left(\frac{2}{5}\right)^2$$

$$1 - 2\left(\frac{4}{25}\right)$$

$$1 - \frac{8}{25}$$

$$\boxed{\frac{17}{25}}$$

e. Find tan y

$$\frac{2}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \boxed{\frac{2\sqrt{21}}{21}}$$

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18) Find $\cos(345^\circ)$.

$\cos(300+45)$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)-\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$

$\boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

19) $\tan^{-1}(-\frac{\sqrt{3}}{3})$

$\boxed{150^\circ \quad 330^\circ}$

20) $\sin^{-1}(-\frac{\sqrt{3}}{2})$

$\boxed{240^\circ \quad 300^\circ}$

21) $\cos^{-1}(-\frac{1}{2})$

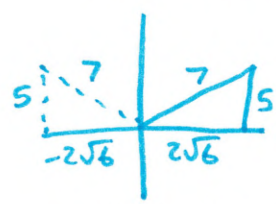
$\boxed{120^\circ \quad 240^\circ}$

22) No calculator: Give the exact value of $\cos\frac{5\pi}{12}$ $\left(\frac{2\pi}{12}+\frac{3\pi}{12}\right)$ $\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

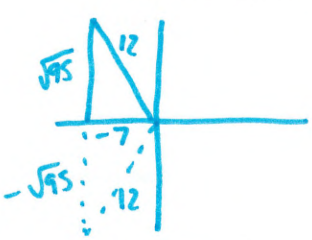
$\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

23) No calculator: Evaluate $\cos(\sin^{-1}\left(\frac{5}{7}\right))$



$\boxed{\pm\frac{2\sqrt{6}}{7}}$

24) Give the exact value for $\sin(\sec^{-1}(-\frac{12}{7}))$



$\boxed{\pm\frac{\sqrt{95}}{12}}$

25) No Calculator: Evaluate $\tan(\sin^{-1}(-\frac{\sqrt{3}}{2}))$

$\boxed{\pm\sqrt{3}}$

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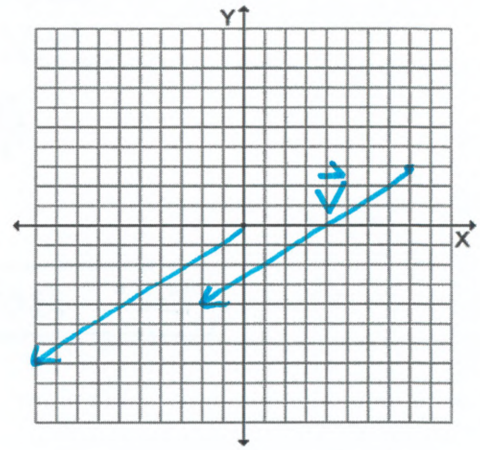
Chapter 8: Vectors and Parametric Equations



-8 -3

26) Using the vector, v, from (8, 3) to (-2, -4), answer the following questions:

a) Draw the vector v. -8 -3

b) Draw the vector in standard position.

c) Write the components of v.

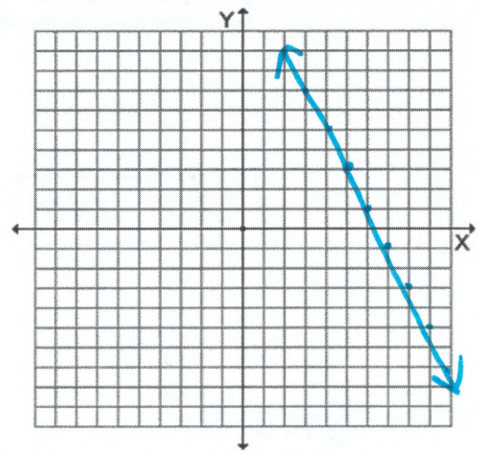
$\langle -10, -7 \rangle$

d) Write v as the sum of unit vectors.

$-10\vec{i} - 7\vec{j}$

e) Write a vector that is orthogonal to v.

$\langle 7, -10 \rangle$ or $\langle -7, 10 \rangle$



27) Graph the line with parametric equations: $x = 5 - 2t$ and $y = -3 + 4t$.

$(5, -3)$

$\langle -2, 4 \rangle \quad m = \frac{4}{-2} = -2$

28) Write the parametric equations for the line through (1, -4) that is parallel to the line with parametric equations $x = 3 - 4t$ and $y = -3 + 7t$.

$\langle -4, 7 \rangle$

$$\boxed{\begin{aligned} x &= 1 - 4t \\ y &= -4 + 7t \end{aligned}}$$

29) Write the parametric equations for the line through (1, -4) that is orthogonal to the line with parametric equations $x = 3 - 4t$ and $y = -3 + 7t$.

$\langle -4, 7 \rangle \quad m = \frac{7}{-4} \quad \perp m = \frac{4}{7} \quad \langle 7, 4 \rangle$

$$\boxed{\begin{aligned} x &= 1 + 7t \\ y &= -4 + 4t \end{aligned}}$$

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30) Find the magnitude and direction of the vector that has initial point (5, -3) and terminal point (6, 1) **Hint**: Put in standard position first.

$-5 \quad +3$ $\qquad$ $-5 \quad +3$

$\langle 1, 4 \rangle$

$|\vec{v}| = \sqrt{1^2+4^2} = \sqrt{17} \approx \boxed{4.12}$

$\tan^{-1}\left(\frac{4}{1}\right) \approx \boxed{75.96^\circ}$

31) Find the measure of the angle between **s** = (-1, 3) and **t** = (-2, -8).

$\cos x = \dfrac{\vec{s} \cdot \vec{t}}{|\vec{s}||\vec{t}|}$

$\vec{s} \cdot \vec{t} = -22$

$|\vec{s}| = \sqrt{10}$

$|\vec{t}| = \sqrt{68}$

$\cos^{-1}\left(\dfrac{-22}{\sqrt{10}\sqrt{68}}\right) = x$

$\boxed{x \approx 147.53^\circ}$

32) Given vector **k** = (-1, 2) and vector **v** = (10, 5), determine whether the two vectors are parallel, perpendicular, or neither.

$m = -2$ $\qquad$ $m = \frac{5}{10} = \frac{1}{2}$

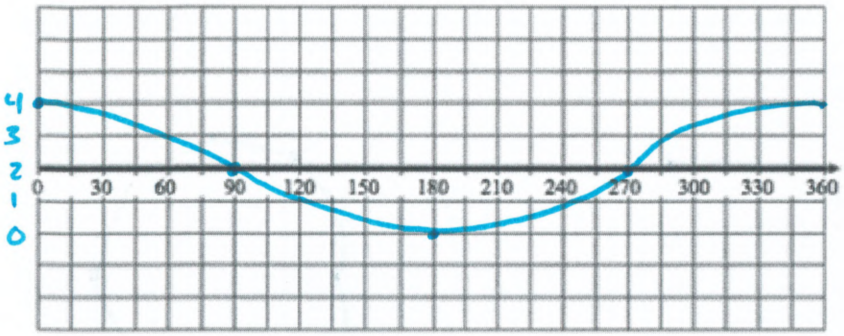
Perpendicular

33) A boat travels at a speed of 75 mph at 26° north of east. It encounters a 10 mph current going 32° south of east. Estimate the resultant speed of the boat.

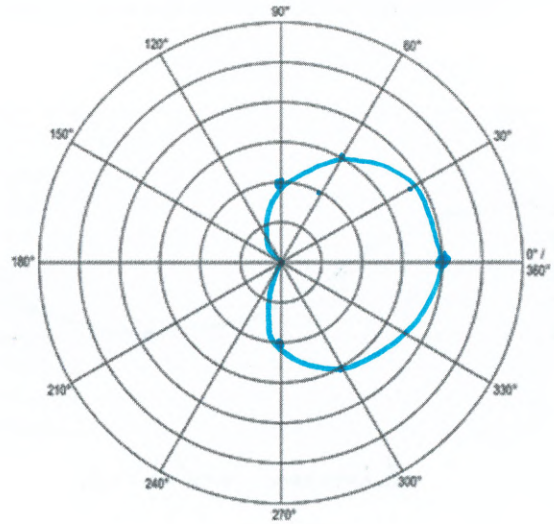
$\langle 67.41, 32.88 \rangle + \langle 8.48, -5.3 \rangle = \langle 75.89, 27.58 \rangle$

$\sqrt{75.89^2 + 27.58^2} \approx \boxed{80.7 \text{ mph}}$

d) $r = 2 + 2\cos(\theta)$



Cardioid



35) Find the 3 other polar coordinates that represent [5, -30°].

[-5, 150°] [5, 330°] [-5, -210°]

36) Given the following polar coordinate, find it's equivalent rectangular coordinate. [-5, 45°].

$x = -5\cos 45$

$y = -5\sin 45$

$\boxed{(-3.54, -3.54)}$

37) Write (-2, -3) in the polar form.

$\sqrt{(-2)^2 + (-3)^2} = 3.61$

$\tan^{-1}\left(\frac{-3}{-2}\right) = 56.3°$

$\boxed{[-3.61, 56.3°]}$

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Chapter 15A: Derivatives

38) The height (in feet) after *t* seconds of a batted ball with initial vertical velocity of 60 ft/sec is given by $h(t) = 60t - 16t^2$.

a. Find the average vertical velocity from time 2 seconds to time 3 seconds (include units).

(2, 56) (3, 36)

$\frac{-20}{1} = \boxed{-20 \text{ ft./sec.}}$

b. Find a formula for the slope of the **secant** line from time *t* to time $t + \Delta t$.

$$\frac{f(t+\Delta t) - f(t)}{\Delta t}$$

c. Find a formula for instantaneous velocity.

$h'(t) = 60 - 32t$

d. What is the instantaneous velocity at $t = 2$? (Include units)

$h'(2) = \boxed{-4 \text{ ft./sec.}}$

e. What is the instantaneous acceleration at $t = 2$? (Include units)

$h''(t) = -32$ $\boxed{-32 \text{ ft./sec.}^2}$

39) Given $f(x) = \dfrac{-8x^2}{\sin x}$, find $f'(x)$.

$$f'(x) = \frac{(\sin x)(-16x) - (-8x^2)(\cos x)}{\sin^2 x}$$

$$= \boxed{\frac{-16x \sin x + 8x^2 \cos x}{\sin^2 x}}$$

40) Use the limit definition of the derivative to calculate $f'(x)$ for $f(x) = 4x + 5$. Show all work.

$$\lim_{\Delta x \to 0} \frac{4(x+\Delta x)+5 - (4x+5)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{4x + 4\Delta x + 5 - 4x - 5}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{4\Delta x}{\Delta x}$$

$$\lim_{\Delta x \to 0} = 4$$

$\boxed{4}$

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41) Write the equation of the tangent line for the function $f(x) = 4x^3 + 5x^2 - 10$ at $x = 2$. Show all work in determining this equation. $(2, 42)$

$f'(x) = 12x^2 + 10x$

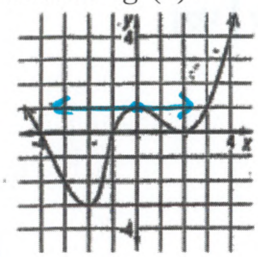
$f'(2) = 68$

$\boxed{y - 42 = 68(x-2)}$

42) Consider the function graphed at the right. Use the geometric definition of the derivative to calculate $g'(0)$.

$g'(0) = 0$

slope of the tangent line at 0.



43) Find a formula to calculate the slope of the line tangent to the function $f(x) = (6x+8)\cos x$.

$f'(x) = (6x+8)(-\sin x) + (\cos x)(6)$

$= \boxed{-6x\sin x - 8\sin x + 6\cos x}$

44) Given $f(x) = \ln(12x + 2x^2)$, find $f'(x)$.

$u = 12x + 2x^2$ $\quad f(u) = \ln u$

$u' = 12 + 4x$ $\quad f'(u) = \frac{1}{u}$

$(12+4x)\left(\frac{1}{u}\right) = \frac{12+4x}{u} = \frac{12+4x}{12x+2x^2} = \frac{2(6+2x)}{2x(6+x)} = \boxed{\frac{6+2x}{6x+x^2}}$

Chapter 15B: Integrals

45) No Calculator: Evaluate each

A) $\int_3^7 (x+3)dx + \int_1^3 (-x)dx$

$\frac{x^2}{2} + 3x \Big|_3^7$ $\quad 45.5 - 13.5 = 32$

$\frac{x^2}{2} \Big|_1^3$ $\quad -4.5 - (-.5) = -4$

$32 + (-4) = \boxed{28}$

B) $\int_0^1 (\sqrt{1-x^2})dx + \int_0^6 (x^2)dx$

$\frac{2(1-x^2)^{3/2}}{3} \Big|_0^1$ $\quad 0 - \frac{2}{3} = -\frac{2}{3}$

$\frac{x^3}{3} \Big|_0^6$ $\quad 72 - 0 = 72$

$-\frac{2}{3} + 72 = \boxed{71\frac{1}{3}}$

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46) For the function $f(x) = x^2 + 1$ Estimate the area bounded by the x-axis, y-axis, f(x) and x = 20 by partitioning the interval from 0 to 20 into 4 equal subintervals of equal length and using $f(z_i)$ where z_i is the value of the right endpoint. Recalculate using z_i being the value of the left endpoint.

Right

X	Y
5	26
10	101
15	226
20	401

$5(26+101+226+401)$

$= \boxed{3770 \text{ un}^2}$

Left

X	Y
0	1
5	26
10	101
15	226

$5(1+26+101+226)$

$= \boxed{1770 \text{ un.}^2}$

47) Given $\int_0^5 (x^2+3)dx - \int_0^4 (x^2+3)dx$.

a) Use the properties of integrals to write the expression as a single integral.

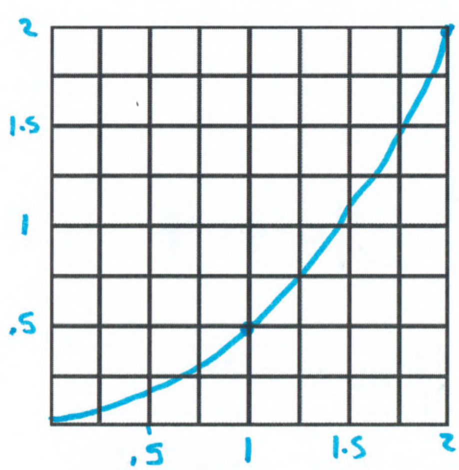
$\int_4^5 (x^2+3)\,dx$

b) Evaluate the integral.

$\frac{x^3}{3} + 3x \Big|_4^5 \quad 56\frac{2}{3} - 33\frac{1}{3} = \boxed{23\frac{1}{3}}$

48) Given the velocity function $f(x) = .5x^2$ on the interval $0 \le x \le 2$:

a) Sketch a picture of the situation.



b) Use Riemann Sums to estimate $\sum_{i=1}^{4} (f(z_i)\Delta x$ where z_i is the right endpoint of the ith subinterval.

X	Y
.5	.125
1	.5
1.5	1.125
2	2

$.5(.125 + .5 + 1.125 + 2)$

$\boxed{1.875 \text{ un.}^2}$

Name________________________________

Chapter 11: Exponential and Logarithmic Functions

22. Simplify each expression:

a. $\sqrt{16x^2y^7} = 4|x||y^3|\sqrt{y}$

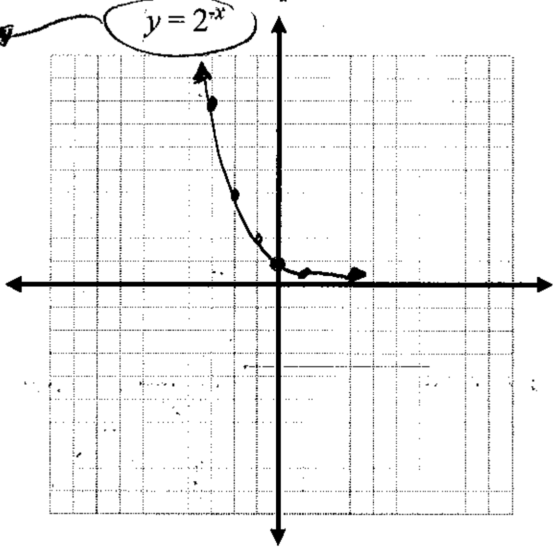
b. $\sqrt[3]{54a^4b^3c^8}$ $\quad 3abc^2\sqrt[3]{2ac^2}$

c. $(3^2c^3d^5)^{\frac{1}{5}} = 3^{\frac{2}{5}}c^{\frac{3}{5}}d \rightsquigarrow d\sqrt[5]{9c^3}$

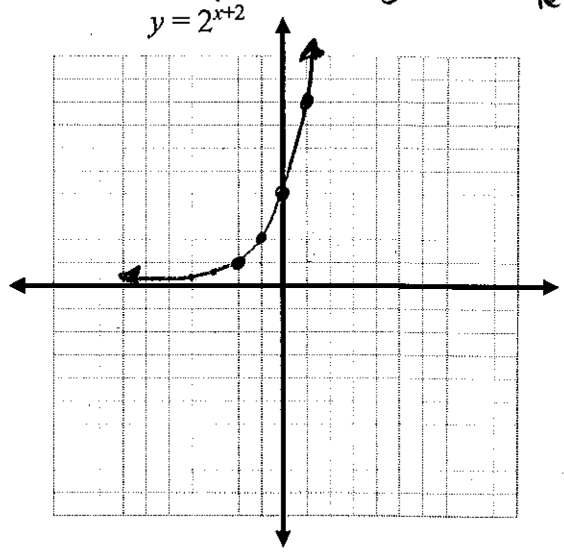
d. $(3x)^2(3x^2)^{-2} = \dfrac{9x^2}{9x^4} = \dfrac{1}{x^2}$

23. Graph each exponential function:

$y = \left(\frac{1}{2}\right)^x$ $\qquad$ $y = 2^{-x}$



$y = 2^{x+2}$ $\qquad$ $y=2^x$ then move left 2



24. A city's population can be modeled by the equation $y = 17492e^{-0.027t}$, where t is the number of years since 1996.

a. What was the city's population in 1996?

17492 people

b. What is the projected population in 2007? $t = 11$

$y = 17492e^{-.027(11)}$

$\approx 12{,}997$ people

Name______________________________

25. Solve each equation:

a. $\log_x 36 = 2$

$x^2 = 36$

$\boxed{x = 6}$

[keep pos. base]

b. $\log_2(2x) = \log_2 27$

$2x = 27$

$\boxed{x = 13.5}$

c. $\log_5 x = \frac{1}{3}\log_5 64 + 2\log_5 3$

$\log_5 x = \underbrace{\log_5 64^{1/3} + \log_5 3^2}$

$\log_5 x = \log_5 4 \cdot 9$

$\boxed{x = 36}$

26. Find the value of each logarithm using the change of base formula.

a. $\log_6 431$

$\frac{\log 431}{\log 6} \approx 3.3856$

b. $\log_{0.5} 78$

$\frac{\log 78}{\log(.5)} \approx -6.2854$

27. Use natural logarithms to solve each equation.

a. $2.3^x = 23.4$

$\log 2.3^x = \log 23.4$

$x \cdot \log 2.3 = \log 23.4$

$\boxed{x \approx 3.7852}$

b. $\log_4 16 = x$

$\frac{\log 16}{\log 4} = \boxed{2}$

c. $5^{x-2} = 2^x$

$\frac{(x-2)\log 5}{\log 5} = \frac{x \cdot \log 2}{\log 5}$

$x - 2 = .4306766x$

$-x \qquad -1x$

$-2 = -.569323x$

$\boxed{x \approx 3.5129}$

d. $\frac{519}{3} = \frac{3e^{0.035t}}{3}$

$173 = e^{0.035t}$

$\frac{\ln 173}{.035} = \frac{0.035t}{.035}$

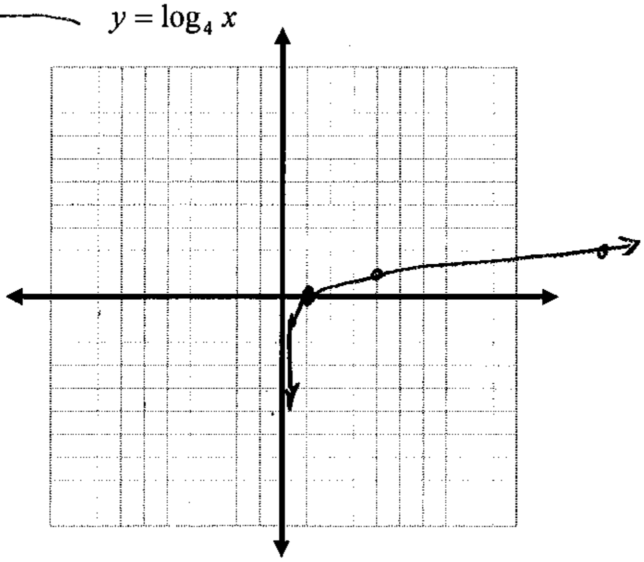
$\boxed{147.2369 \approx t}$

28. Graph each logarithmic function:

$y = \log_4 x$

$4^y = x$

x	y
$\frac{1}{4}$	-1
1	0
4	1
16	2



$y = \log_2 x$

$2^y = x$

x	y
$\frac{1}{2}$	-1
1	0
2	1
4	2